Evapotranspiration – Equations

Penman – Monteith (1965)

$$\lambda ET = \frac{\Delta(R_n - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a}}{\Delta + \gamma(1 + \frac{r_s}{r_a})} = \frac{\Delta(Rn - G) + \rho_a c_p \frac{VPD}{r_a}}{\Delta + \gamma(1 + \frac{r_s}{r_a})} \quad (1)$$

Where:

λ is latent heat of vaporization of water (MJ kg^{-1});

ET is evapotranspiration (mm d^{-1});

R_n is net radiation at the crop surface (MJ m^{-2} d^{-1});

G is soil heat flux (MJ m^{-2} d^{-1});

ρ_a is air average density at constant pressure (kg m^{-3});

c_p is air specific heat (MJ kg^{-1} $°C^{-1}$);

e_s is saturation vapor pressure (kPa);

e_a is actual vapor pressure (kPa);

$(e_s – e_a)$ is saturation vapor pressure deficit (kPa);

VPD is vapor pressure deficit (kPa);

Δ is slope vapor pressure curve (kPa $°C^{-1}$);

γ is psychrometric constant (kPa $°C^{-1}$);

r_s is surface resistance (s m^{-1});

r_a is aerodynamic resistance (s m^{-1}).

Penman – Monteith (FAO 56)

$$\mathrm{ETo} = \frac{0.408\Delta(\mathrm{Rn} - \mathrm{G}) + \gamma \frac{900}{T_{mean} + 273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34 u_2)} \tag{2}$$

Where:

ET_o is reference evapotranspiration (mm day^{-1});

R_n is net radiation at the crop surface (MJ m^{-2} day^{-1});

G is soil heat flux density (MJ m^{-2} day^{-1});

T is mean daily air temperature at 2 m height (m s^{-1});

u_2 is wind speed at 2 m height (m s^{-1});

e_s is saturation vapor pressure (kPa);

e_a is real vapor pressure (kPa);

$e_s - e_a$ is saturation vapor pressure deficit (kPa);

Δ is slope vapor pressure curve (kPa °C^{-1});

γ is psychrometric constant (kPa °C^{-1}).

Considering the following restructure from Penman-Monteith (1965) equation:

$$\lambda ET = \frac{\Delta(R_n - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a}}{\Delta + \gamma(1 + \frac{r_s}{r_a})} \tag{3}$$

$$\lambda ET \left(\Delta + \gamma(1 + \frac{r_s}{r_a})\right) = \Delta(R_n - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a} \tag{4}$$

$$ET \left(\Delta + \gamma(1 + \frac{r_s}{r_a})\right) = \frac{\Delta(R_n - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a}}{\lambda} \tag{5}$$

$$ET \left(\Delta + \gamma(1 + \frac{r_s}{r_a})\right) = \frac{\Delta(R_n - G)}{\lambda} + \frac{\rho_a c_p \frac{(e_s - e_a)}{r_a}}{\lambda} \tag{6}$$

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \frac{\rho_a c_p \frac{(e_s - e_a)}{r_a}}{\lambda}}{\Delta + \gamma(1 + \frac{r_s}{r_a})} \tag{7}$$

Considering the following approach of all parameters from Penman-Monteith (FAO56) equation:

"A hypothetical reference crop with an assumed crop height of 0.12 m, a fixed surface resistance of 70 s m^{-1} and an albedo of 0.23." (Allen et al. 1998).

$$r_a = \frac{208}{u_2} \tag{8}$$

$$r_s = 70 \tag{9}$$

Replace the parameters from reference crop in PM equation:

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \frac{\rho_a c_p \frac{(e_s - e_a)}{r_a}}{\lambda}}{\Delta + \gamma(1 + \frac{r_s}{r_a})} \tag{10}$$

Considering:

$$\rho_a = \frac{P}{T_{kv} R_d} \tag{11}$$

$$c_p = \frac{\gamma \varepsilon \lambda}{P} \tag{12}$$

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \frac{\frac{P}{T_{kv} R_d} \frac{\gamma\varepsilon\lambda}{P} \frac{(e_s - e_a)}{\frac{208}{u_2}}}{\lambda}}{\Delta + \gamma(1 + \frac{70}{\frac{208}{u_2}})} \tag{13}$$

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \frac{\frac{\gamma\varepsilon\lambda}{T_{kv} R_d} \frac{u_2}{208} (e_s - e_a)}{\lambda}}{\Delta + \gamma(1 + 0.34 u_2)} \tag{14}$$

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \frac{\gamma\varepsilon}{T_{kv} R_d} \frac{u_2}{208} (e_s - e_a)}{\Delta + \gamma(1 + 0.34 u_2)} \tag{15}$$

Considering:

ε is ratio molecular weight of water vapor/dry air = 0.622.

T_{kv} is virtual temperature = 1.01 (T + 273) ~ (T + 273).

R_d is universal gas constant specific for dry air = 0.287 kJ kg^{-1} K^{-1}.

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \frac{\gamma 0.622}{(T + 273)0.287}\frac{u_2}{208}\,(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \tag{16}$$

<u>Considering a time of one day (86400 seconds):</u>

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + 86400\frac{\gamma 0.622}{(T + 273)0.287}\frac{u_2}{208}\,(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \tag{17}$$

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \gamma\frac{900}{(T + 273)}u_2\,(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \tag{18}$$

<u>Considering the latent heat vaporization (λ) equal = 2.45 MJ kg^{-1}</u>

"As λ varies only slightly over normal temperature ranges a single value of 2.45 MJ kg^{-1} is taken in the simplification of the FAO Penman-Monteith equation. This is the latent heat flux for an air temperature of about 20 °C" (Allen et al. 1998).

$$ET = \frac{0.408\Delta(R_n - G) + \gamma\frac{900}{(T + 273)}u_2\,(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \tag{19}$$

Considering the time of 1 hour (3600 seconds):

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + 3600\frac{\gamma 0.622}{(T + 273)0.287}\frac{u_2}{208}(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \qquad (20)$$

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \gamma\frac{37.51}{(T + 273)}u_2\ (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \qquad (21)$$

Considering the time of 30 minutes (1800 seconds):

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + 1800\frac{\gamma 0.622}{(T + 273)0.287}\frac{u_2}{208}(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \qquad (22)$$

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \gamma\frac{18.75}{(T + 273)}u_2\ (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \qquad (23)$$

Considering the time of 15 minutes (900 seconds):

$$ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + 900\frac{\gamma 0.622}{(T + 273)0.287}\frac{u_2}{208}(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \qquad (24)$$

$$\boxed{ET = \frac{\frac{\Delta(R_n - G)}{\lambda} + \gamma\frac{9.37}{(T + 273)}u_2\ (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \qquad (25)}$$

Reference:

Allen RG, Pereira LS, Raes D, Smith M (1998). Crop evapotranspiration - Guidelines for computing crop water requirements - FAO Irrigation and drainage paper 56. FAO, Rome, 300(9), D05109.

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