

Package ‘fbardl’

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Type Package

Title Fourier Bootstrap ARDL Cointegration Test

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Description Implements the Fourier Bootstrap Autoregressive Distributed Lag (FBARDL) bounds testing approach for cointegration analysis. Combines the Pesaran, Shin and Smith (2001) <[doi:10.1002/jae.616](https://doi.org/10.1002/jae.616)> ARDL bounds testing framework with Fourier terms to capture smooth structural breaks, as in Yilanci, Bozoklu and Gorus (2020) <[doi:10.1016/j.scs.2020.102035](https://doi.org/10.1016/j.scs.2020.102035)>, and recursive bootstrap critical values following McNown, Sam and Goh (2018) <[doi:10.1080/00036846.2017.1366643](https://doi.org/10.1080/00036846.2017.1366643)> and Bertelli, Vacca and Zoia (2022) <[doi:10.1016/j.econmod.2022.105987](https://doi.org/10.1016/j.econmod.2022.105987)>, with the Fourier frequency and the lags selected again in every bootstrap replication (a package choice); finite-sample bounds test critical values from Kripfganz and Schneider (2020) <[doi:10.1111/obes.12377](https://doi.org/10.1111/obes.12377)> for models without Fourier terms. Features include lag selection via AIC/BIC, Fourier frequency selection by minimum SSR, long-run and short-run coefficient estimation and diagnostic tests.

License GPL-3

Encoding UTF-8

Depends R (>= 3.5.0)

Imports stats

Suggests testthat (>= 3.0.0), knitr, rmarkdown

LazyData true

Config/testthat/edition 3

RoxygenNote 7.3.3

URL <https://github.com/muhammedalkhalaf/fbardl>

BugReports <https://github.com/muhammedalkhalaf/fbardl/issues>

NeedsCompilation no

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Description

ARDL bounds tests for cointegration (Pesaran, Shin and Smith, 2001) with optional Fourier terms for smooth breaks and bootstrap critical values (McNown, Sam and Goh, 2018; Bertelli, Vacca and Zoia, 2022).

Usage

```
fbardl(  
  formula,  
  data,  
  type = c("fbardl_bvz", "fbardl_mcnown", "fardl"),  
  maxlag = 4,  
  maxk = 5,  
  ic = c("aic", "bic"),  
  case = 3,  
  reps = 999,  
  fourier = TRUE,  
  level = 0.95,  
  horizon = 20,  
  unconditional = FALSE,  
  kgrid = c("integer", "fractional"),  
  kstar = NULL,  
  lags = NULL,  
  seed = NULL  
)
```

Arguments

- | | |
|---------|--|
| formula | A formula of the form $y \sim x_1 + x_2 + \dots$ specifying the dependent and independent variables. |
| data | A data frame containing the time series variables. |

type	Character string specifying the test type: • "fbardl_bvz" (default): bootstrap with separate nulls (Bertelli, Vacca and Zoia, 2022) • "fbardl_mcnown": bootstrap with the null of the overall F test for all statistics (McNown, Sam and Goh, 2018) • "fardl": Kripfganz and Schneider (2020) bounds; valid only without Fourier terms (see Details)
maxlag	Integer. Maximum lag order for grid search (default: 4).
maxk	Numeric. Maximum Fourier frequency (default: 5).
ic	Character string. Information criterion for lag selection: "aic" (default) or "bic".
case	Integer. PSS case specification (2, 3, 4, or 5). Default is 3 (unrestricted intercept, no trend).
reps	Integer. Number of bootstrap replications (default: 999).
fourier	Logical. Whether to include Fourier terms (default: TRUE).
level	Numeric. Confidence level of the long-run coefficient intervals (default: 0.95). Test decisions are made at the 5% level.
horizon	Not used; kept for compatibility with earlier versions (no dynamic multipliers are computed).
unconditional	Logical. If TRUE, the contemporaneous differences of the regressors are excluded from the estimated model, as in the simulation model of McNown, Sam and Goh (2018, eq. 12). Default FALSE.
kgrid	Grid of candidate Fourier frequencies when k^* is selected: "integer" (default), $k = 1, \dots, \text{maxk}$, or "fractional", $k = 0.1, 0.2, \dots, \text{maxk}$ (the only grid of fbardl 1.1.0 and earlier). See Details.
kstar	Optional Fourier frequency fixed by the user in advance (requires fourier = TRUE). It is then not selected, neither on the data nor in the bootstrap.
lags	Optional list with elements p (number of lagged differences of y) and q (highest lag of the differences of each regressor, one value per regressor or a single value for all) fixing the lags in advance. They are then not selected, neither on the data nor in the bootstrap.
seed	Optional integer seed for the bootstrap; the random number generator state of the session is restored afterwards. With NULL (default) the session's generator is used.

Details

Model. The equilibrium-correction model regresses Δy_t on y_{t-1} , x_{t-1} , p lagged differences of y , the differences of each regressor at lags 0 (1 if unconditional = TRUE) to q_j , the Fourier terms $\sin(2\pi k^* t/T)$ and $\cos(2\pi k^* t/T)$, a constant and, in PSS cases 4 and 5, a linear trend. The overall F test (Fov) restricts the lagged levels, together with the intercept in case 2 and the trend in case 4; t is the t statistic on y_{t-1} and Find the F test on the lagged regressors.

Selection. Step 1: unless kstar is given, k^* minimises the sum of squared residuals of the model with every lag at maxlag. With kgrid = "integer" the candidates are $1, \dots, \text{maxk}$, the integer

frequencies chosen by minimum SSR in Enders and Lee (2012); with `kgrid = "fractional"` they are 0.1, 0.2, ..., `maxk`. The integer default, and whether the fractional grid is appropriate, are package choices pending verification against Yilanci, Bozoklu and Gorus (2020) and Omay (2015). Step 2: unless lags is given, p (1 to `maxlag`) and each q_j (0 to `maxlag`) minimise the AIC or BIC with k^* fixed.

Bounds (`type = "fardl"`). Without Fourier terms (`fourier = FALSE`) `Fov` and `t` are compared with the finite-sample critical values and approximate p-values of Kripfganz and Schneider (2020), for the sample size, the number of regressors and the number of short-run coefficients, and the 5% bounds decision is reported. With Fourier terms no valid bounds exist: no p-values and no decision are given, and the bounds printed are the bounds for the model without Fourier terms (short-run coefficients counted without the Fourier terms); they are not valid with Fourier terms. Use a bootstrap type for inference. `Find` has no tabulated distribution.

Bootstrap ("`fbardl_bvz`", "`fbardl_mcnown`"). The bootstrap uses the recursive engine shared with the package `ardlverse` (file `R/ardl_boot_engine.R`, engine version 1.1.0). Once, on the data: the restricted equation for Δy (under the null of each statistic) and the equation for Δx are estimated with the selected (or fixed) k^* , p and q , and their residuals are saved. "`fbardl_bvz`" follows Bertelli, Vacca and Zoia (2022, Section 3): separate nulls for `Fov`, `t` and `Find`, the marginal VECM for Δx (x_{t-1} and p lags of Δy and Δx), and initial values drawn as a random block of the data. "`fbardl_mcnown`" uses the null of `Fov` for all statistics (McNown, Sam and Goh, 2018, Steps 1 to 8); MSG write the equation for y without the contemporaneous differences (their eq. 12), which is the form used with `unconditional = TRUE`; with the default conditional ECM, applying their joint null to it is a package choice. The Δx equation is unrestricted (it also contains y_{t-1}), and the initial values are the first observations of the data (with "`fbardl_bvz`" the levels in the random block start the recursion). In each replication: (1) the residual pairs are resampled with replacement and recentred (after each draw for "`fbardl_bvz`"; once, before resampling, for "`fbardl_mcnown`"); (2) x^* and y^* are generated recursively from the estimated equations, with the Fourier terms (at the data's k^*) and the trend held fixed; (3) k^* is selected again on the bootstrap sample by Step 1 (unless `kstar` is given or `fourier = FALSE`) and the lags by Step 2 (unless `lags` is given); (4) the model is estimated with the selected specification and `Fov`, `t` and `Find` are computed exactly as on the data. Repeating the selection in each replication is a package choice (BVZ step 5(c) re-estimates the unrestricted model but does not discuss re-selection; McNown, Sam and Goh do not discuss it either); it makes the bootstrap distribution account for the data-based selection. In the package's Monte Carlo experiments ($n = 100$, independent random walks, `maxlag = 1`, 200 samples, 199 replications, 5% level), the default call gave sizes 0.075 (`Fov`), 0.135 (`t`), 0.110 (`Find`) and 0.040 for the combined decision; holding k^* and the lags fixed at their data-selected values (`fbardl` 1.1.0) gave 0.375, 0.420 and 0.270. With `unconditional = TRUE` the generating equation for Δy also omits the contemporaneous differences of the regressors, so the bootstrap model is the estimated model in both forms. P-values are the shares of bootstrap statistics at least as extreme as the observed one; critical values are order statistics (MSG eqs. 15-16, BVZ eqs. 24-25). The decision (5% level) requires `Fov`, `t` and `Find` to reject. `Fov` rejecting and `t` not is the degenerate case of the first type (whatever `Find` gives); `Fov` and `t` rejecting and `Find` not is the degenerate case of the second type (terminology of Bertelli, Vacca and Zoia 2022, eqs. 8-9; McNown, Sam and Goh 2018 number the two degenerate cases the other way). The same labels are used for both bootstrap types. Failed replications are set to NA and counted. The run time grows with `reps`, `maxlag`, the number of regressors and the frequency grid, because the selection is repeated in every replication; fixing `kstar` and `lags` removes it.

Value

An object of class "fbardl" containing:

coefficients Named vector of estimated coefficients

std.errors Standard errors of coefficients

t.values t-statistics

p.values p-values

long.run Long-run coefficient estimates with standard errors

short.run Short-run coefficient estimates

ecm.coef Error correction coefficient (speed of adjustment)

best.p Selected (or fixed) lag order for the dependent variable

best.q Selected (or fixed) lag orders for the independent variables

best.kstar Selected (or fixed) Fourier frequency

kgrid, kstar.fixed, lags.fixed, ssr.by.k Frequency grid used, whether k* and the lags were fixed by the user, and the SSR of each candidate frequency

F.overall F-statistic for overall cointegration test

t.dependent t-statistic on lagged dependent variable

F.independent F-statistic on lagged independent variables

cointegration Cointegration test results: p-values, critical values, decision (text) and decision.code; for the bootstrap types also the bootstrap distributions (Fov.boot, t.boot, Find.boot), the numbers of valid and failed replications, reselect (what was selected again in each replication) and dgpcheck

diagnostics Diagnostic test results

model.fit Model fit statistics (R2, AIC, BIC, etc.)

residuals Model residuals

fitted.values Fitted values

nobs Number of observations

call The matched call

References

- Bertelli, S., Vacca, G. and Zoia, M. (2022). Bootstrap cointegration tests in ARDL models. *Economic Modelling*, 116, 105987. doi:[10.1016/j.econmod.2022.105987](https://doi.org/10.1016/j.econmod.2022.105987)
- Enders, W. and Lee, J. (2012). A unit root test using a Fourier series to approximate smooth breaks. *Oxford Bulletin of Economics and Statistics*, 74(4), 574-599. doi:[10.1111/j.14680084.2011.00662.x](https://doi.org/10.1111/j.14680084.2011.00662.x)
- Kripfganz, S. and Schneider, D. C. (2020). Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models. *Oxford Bulletin of Economics and Statistics*, 82(6), 1456-1481. doi:[10.1111/obes.12377](https://doi.org/10.1111/obes.12377)
- McNown, R., Sam, C. Y. and Goh, S. K. (2018). Bootstrapping the autoregressive distributed lag test for cointegration. *Applied Economics*, 50(13), 1509-1521. doi:[10.1080/00036846.2017.1366643](https://doi.org/10.1080/00036846.2017.1366643)

Omay, T. (2015). Fractional frequency flexible Fourier form to approximate smooth breaks in unit root testing. *Economics Letters*, 134, 123-126. doi:10.1016/j.econlet.2015.07.010

Pesaran, M. H., Shin, Y. and Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics*, 16(3), 289-326. doi:10.1002/jae.616

Yilanci, V., Bozoklu, S. and Gorus, M. S. (2020). Are BRICS countries pollution havens? Evidence from a bootstrap ARDL bounds testing approach with a Fourier function. *Sustainable Cities and Society*, 55, 102035. doi:10.1016/j.scs.2020.102035

Examples

```
data(fbardl_data)

# Bootstrap (Bertelli, Vacca and Zoia), k* and lags selected again in
# each replication; few replications to keep the example short
result <- fbardl(y ~ x1 + x2, data = fbardl_data, maxlag = 2, maxk = 3,
                reps = 49, seed = 1)

result

# McNown, Sam and Goh bootstrap with k* and lags fixed in advance
result_msg <- fbardl(y ~ x1 + x2, data = fbardl_data,
                    type = "fbardl_mcnown", kstar = 1,
                    lags = list(p = 1, q = c(1, 1)), reps = 99, seed = 1)

summary(result_msg)

# Bounds test without Fourier terms
result_bounds <- fbardl(y ~ x1 + x2, data = fbardl_data, type = "fardl",
                       fourier = FALSE)
```

fbardl_data

Example Data for Fourier Bootstrap ARDL Analysis

Description

A simulated time series dataset suitable for demonstrating the Fourier Bootstrap ARDL cointegration testing procedure. The data contains a dependent variable y and two independent variables x_1 and x_2 with a cointegrating relationship and structural breaks.

Usage

```
fbardl_data
```

Format

A data frame with 150 observations and 3 variables:

- y** Dependent variable (simulated I(1) series)
- x1** First independent variable (simulated I(1) series)
- x2** Second independent variable (simulated I(1) series)

Details

The data is generated from a data-generating process (DGP) that includes:

- A long-run cointegrating relationship: $y_t = 2 + 0.8x_{1t} - 0.5x_{2t} + u_t$
- Short-run dynamics with AR(1) errors
- A structural break modeled by Fourier terms
- Error correction mechanism with adjustment speed of -0.3

This dataset is designed to produce clear cointegration test results when analyzed with the [fbardl](#) function.

Source

Simulated data for package demonstration.

See Also

[fbardl](#)

Examples

```
data(fbardl_data)
head(fbardl_data)
summary(fbardl_data)

# Plot the series
ts.plot(ts(fbardl_data), col = 1:3, lty = 1:3)
legend("topleft", colnames(fbardl_data), col = 1:3, lty = 1:3)
```

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