# Package 'gamlss.dist'

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**Title** Distributions for Generalized Additive Models for Location Scale and Shape

**Version** 6.1-11

**Date** 2026-08-10

**Description** A set of distributions which can be used for modelling the response variables in Generalized Additive Models for Location Scale and Shape, Rigby and Stasinopoulos (2005), <doi:10.1111/j.1467-9876.2005.00510.x>. The distributions can be continuous, discrete or mixed distributions. Extra distributions can be created, by transforming, any continuous distribution defined on the real line, to a distribution defined on ranges 0 to infinity or 0 to 1, by using a 'log' or a 'logit' transformation respectively.

**License** GPL-2 | GPL-3

**URL** https://www.gamlss.com/

**BugReports** https://github.com/gamlss-dev/gamlss.dist/issues

**Depends** R (>= 3.5.0)

**Imports** MASS, graphics, stats, methods, grDevices

**Suggests** distributions3 (>= 0.2.1)

**Encoding** UTF-8

**NeedsCompilation** yes

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`gamlss.dist-package` *Distributions for Generalized Additive Models for Location Scale and Shape*

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## Description

A set of distributions which can be used for modelling the response variables in Generalized Additive Models for Location Scale and Shape, Rigby and Stasinopoulos (2005), <doi:10.1111/j.1467-9876.2005.00510.x>. The distributions can be continuous, discrete or mixed distributions. Extra distributions can be created, by transforming, any continuous distribution defined on the real line, to a distribution defined on ranges 0 to infinity or 0 to 1, by using a 'log' or a 'logit' transformation respectively.

## Details

The DESCRIPTION file:

| | |
|---|---|
| Package: | gamlss.dist |
| Title: | Distributions for Generalized Additive Models for Location Scale and Shape |
| Version: | 6.1-11 |
| Date: | 2026-08-10 |
| Authors@R: | c(person("Mikis", "Stasinopoulos", role = c("aut", "cre", "cph"), email = "d.stasinopoulos@gre.ac.uk", comme |
| Description: | A set of distributions which can be used for modelling the response variables in Generalized Additive Models f |
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| URL: | https://www.gamlss.com/ |
| BugReports: | https://github.com/gamlss-dev/gamlss.dist/issues |
| Depends: | R (>= 3.5.0) |
| Imports: | MASS, graphics, stats, methods, grDevices |
| Suggests: | distributions3 (>= 0.2.1) |
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Index of help topics:

| | |
|---|---|
| BB | Beta Binomial Distribution For Fitting a GAMLSS Model |
| BCCG | Box-Cox Cole and Green distribution (or Box-Cox normal) for fitting a GAMLSS |
| BCPE | Box-Cox Power Exponential distribution for fitting a GAMLSS |
| BCT | Box-Cox t distribution for fitting a GAMLSS |
| BE | The beta distribution for fitting a GAMLSS |

| | |
|---|---|
| BEINF | The beta inflated distribution for fitting a GAMLSS |
| BEOI | The one-inflated beta distribution for fitting a GAMLSS |
| BEZI | The zero-inflated beta distribution for fitting a GAMLSS |
| BI | Binomial distribution for fitting a GAMLSS |
| BNB | Beta Negative Binomial distribution for fitting a GAMLSS |
| DBI | The Double binomial distribution |
| DBURR12 | The Discrete Burr type XII distribution for fitting a GAMLSS model |
| DEL | The Delaporte distribution for fitting a GAMLSS model |
| DPO | The Double Poisson distribution |
| EGB2 | The exponential generalized Beta type 2 distribution for fitting a GAMLSS |
| EXP | Exponential distribution for fitting a GAMLSS |
| GA | Gamma distribution for fitting a GAMLSS |
| GAF | The Gamma distribution family |
| GAMLSS | Create a GAMLSS Distribution |
| GB1 | The generalized Beta type 1 distribution for fitting a GAMLSS |
| GB2 | The generalized Beta type 2 and generalized Pareto distributions for fitting a GAMLSS |
| GEOM | Geometric distribution for fitting a GAMLSS model |
| GG | Generalized Gamma distribution for fitting a GAMLSS |
| GIG | Generalized Inverse Gaussian distribution for fitting a GAMLSS |
| GPO | The generalised Poisson distribution |
| GT | The generalized t distribution for fitting a GAMLSS |
| GU | The Gumbel distribution for fitting a GAMLSS |
| IG | Inverse Gaussian distribution for fitting a GAMLSS |
| IGAMMA | Inverse Gamma distribution for fitting a GAMLSS |
| JSU | The Johnson's Su distribution for fitting a GAMLSS |
| JSUo | The original Johnson's Su distribution for fitting a GAMLSS |
| LG | Logarithmic and zero adjusted logarithmic distributions for fitting a GAMLSS model |
| LNO | Log Normal distribution for fitting in GAMLSS |
| LO | Logistic distribution for fitting a GAMLSS |
| LOGITNO | Logit Normal distribution for fitting in GAMLSS |
| LQNO | Normal distribution with a specific mean and |

| | |
|---|---|
| | variance relationship for fitting a GAMLSS model |
| MN3 | Multinomial distribution in GAMLSS |
| NBF | Negative Binomial Family distribution for fitting a GAMLSS |
| NBI | Negative Binomial type I distribution for fitting a GAMLSS |
| NBII | Negative Binomial type II distribution for fitting a GAMLSS |
| NET | Normal Exponential t distribution (NET) for fitting a GAMLSS |
| NO | Normal distribution for fitting a GAMLSS |
| NO2 | Normal distribution (with variance as sigma parameter) for fitting a GAMLSS |
| NOF | Normal distribution family for fitting a GAMLSS |
| PARETO2 | Pareto distributions for fitting in GAMLSS |
| PE | Power Exponential distribution for fitting a GAMLSS |
| PIG | The Poisson-inverse Gaussian distribution for fitting a GAMLSS model |
| PO | Poisson distribution for fitting a GAMLSS model |
| RG | The Reverse Gumbel distribution for fitting a GAMLSS |
| RGE | Reverse generalized extreme family distribution for fitting a GAMLSS |
| SEP | The Skew Power exponential (SEP) distribution for fitting a GAMLSS |
| SEP1 | The Skew exponential power type 1-4 distribution for fitting a GAMLSS |
| SHASH | The Sinh-Arcsinh (SHASH) distribution for fitting a GAMLSS |
| SI | The Sichel dustribution for fitting a GAMLSS model |
| SICHEL | The Sichel distribution for fitting a GAMLSS model |
| SIMPLEX | The simplex distribution for fitting a GAMLSS |
| SN1 | Skew Normal Type 1 distribution for fitting a GAMLSS |
| SN2 | Skew Normal Type 2 distribution for fitting a GAMLSS |
| ST1 | The skew t distributions, type 1 to 5 |
| TF | t family distribution for fitting a GAMLSS |
| WARING | Waring distribution for fitting a GAMLSS model |
| WEI | Weibull distribution for fitting a GAMLSS |
| WEI2 | A specific parameterization of the Weibull distribution for fitting a GAMLSS |
| WEI3 | A specific parameterization of the Weibull distribution for fitting a GAMLSS |

| | |
|---|---|
| YULE | Yule distribution for fitting a GAMLSS model |
| ZABB | Zero inflated and zero adjusted Binomial distribution for fitting in GAMLSS |
| ZABI | Zero inflated and zero adjusted Binomial distribution for fitting in GAMLSS |
| ZAGA | The zero adjusted Gamma distribution for fitting a GAMLSS model |
| ZAIG | The zero adjusted Inverse Gaussian distribution for fitting a GAMLSS model |
| ZANBI | Zero inflated and zero adjusted negative binomial distributions for fitting a GAMLSS model |
| ZAP | Zero adjusted poisson distribution for fitting a GAMLSS model |
| ZIP | Zero inflated poisson distribution for fitting a GAMLSS model |
| ZIP2 | Zero inflated poisson distribution for fitting a GAMLSS model |
| ZIPF | The zipf and zero adjusted zipf distributions for fitting a GAMLSS model |
| checklink | Set the Right Link Function for Specified Parameter and Distribution |
| count_1_31 | A set of functions to plot gamlss.family distributions |
| exGAUS | The ex-Gaussian distribution |
| flexDist | Non-parametric pdf from limited information data |
| gamlss.dist-package | Distributions for Generalized Additive Models for Location Scale and Shape |
| gamlss.family | Family Objects for fitting a GAMLSS model |
| gen.Family | Functions to generate log and logit distributions from existing continuous gamlss.family distributions |
| hazardFun | Hazard functions for gamlss.family distributions |
| make.link.gamlss | Create a Link for GAMLSS families |
| momentSK | Sample and theoretical Moment and Centile Skewness and Kurtosis Functions |

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## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
# pdf plot
plot(function(y) dSICHEL(y, mu=10, sigma = 0.1 , nu=1 ),
              from=0, to=30, n=30+1, type="h")
# cdf plot
PPP <- par(mfrow=c(2,1))
plot(function(y) pSICHEL(y, mu=10, sigma =0.1, nu=1 ),
             from=0, to=30, n=30+1, type="h") # cdf
cdf<-pSICHEL(0:30, mu=10, sigma=0.1, nu=1)
sfun1  <- stepfun(1:30, cdf, f = 0)
plot(sfun1, xlim=c(0,30), main="cdf(x)")
par(PPP)
```

---

BB *Beta Binomial Distribution For Fitting a GAMLSS Model*

---

## Description

This function defines the beta binomial distribution, a two parameter distribution, for a gamlss.family object to be used in a GAMLSS fitting using the function gamlss()

## Usage

```
BB(mu.link = "logit", sigma.link = "log")
dBB(x, mu = 0.5, sigma = 1, bd = 10, log = FALSE)
pBB(q, mu = 0.5, sigma = 1, bd = 10, lower.tail = TRUE,
      log.p = FALSE)
qBB(p, mu = 0.5, sigma = 1, bd = 10, lower.tail = TRUE,
       log.p = FALSE)
rBB(n, mu = 0.5, sigma = 1, bd = 10, ...)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog"'(complementary log-log) |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" and "sqrt" |
| mu | vector of positive probabilities |
| sigma | the dispersion parameter |
| bd | vector of binomial denominators |
| p | vector of probabilities |
| x, q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| ... | further arguments |

## Details

Definition file for beta binomial distribution.

$$f(y|\mu,\sigma) = \frac{\Gamma(n+1)}{\Gamma(y+1)\Gamma(n-y+1)} \frac{\Gamma(\frac{1}{\sigma})\Gamma(y+\frac{\mu}{\sigma})\Gamma[n+\frac{(1-\mu)}{\sigma}-y]}{\Gamma(n+\frac{1}{\sigma})\Gamma(\frac{\mu}{\sigma})\Gamma(\frac{1-\mu}{\sigma})}$$

for $y = 0, 1, 2, \ldots, n$, $0 < \mu < 1$ and $\sigma > 0$, see pp. 523-524 of Rigby *et al.* (2019). . For $\mu = 0.5$ and $\sigma = 0.5$ the distribution is uniform.

## Value

Returns a gamlss.family object which can be used to fit a Beta Binomial distribution in the gamlss() function.

## Warning

The functions pBB and qBB are calculated using a laborious procedure so they are relatively slow.

## Note

The response variable should be a matrix containing two columns, the first with the count of successes and the second with the count of failures. The parameter mu represents a probability parameter with limits $0 < \mu < 1$. $n\mu$ is the mean of the distribution where n is the binomial denominator. $\{n\mu(1-\mu)[1+(n-1)\sigma/(\sigma+1)]\}^{0.5}$ is the standard deviation of the Beta Binomial distribution. Hence $\sigma$ is a dispersion type parameter

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Kalliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BI,

## Examples

```
# BB()# gives information about the default links for the Beta Binomial distribution
#plot the pdf
plot(function(y) dBB(y, mu = .5, sigma = 1, bd =40), from=0, to=40, n=40+1, type="h")
#calculate the cdf and plotting it
ppBB <- pBB(seq(from=0, to=40), mu=.2 , sigma=3, bd=40)
plot(0:40,ppBB, type="h")
#calculating quantiles and plotting them
qqBB <- qBB(ppBB, mu=.2 , sigma=3, bd=40)
plot(qqBB~ ppBB)
# when the argument fast is useful
p <- pBB(c(0,1,2,3,4,5), mu=.01 , sigma=1, bd=5)
qBB(p, mu=.01 , sigma=1, bd=5)
#  0 1 1 2 3 5
qBB(p, mu=.01 , sigma=1, bd=5)
#  0 1 2 3 4 5
# generate random sample
tN <- table(Ni <- rBB(1000, mu=.2, sigma=1, bd=20))
r <- barplot(tN, col='lightblue')
# fitting a model
```

```
# library(gamlss)
#data(aep)
# fits a Beta-Binomial model
#h<-gamlss(y~ward+loglos+year, sigma.formula=~year+ward, family=BB, data=aep)
```

---

BCCG *Box-Cox Cole and Green distribution (or Box-Cox normal) for fitting a GAMLSS*

---

## Description

The function BCCG defines the Box-Cox Cole and Green distribution (Box-Cox normal), a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dBCCG, pBCCG, qBCCG and rBCCG define the density, distribution function, quantile function and random generation for the specific parameterization of the Box-Cox Cole and Green distribution. [The function BCCGuntr() is the original version of the function suitable only for the untruncated Box-Cox Cole and Green distribution See Cole and Green (1992) and Rigby and Stasinopoulos (2003a, 2003b) for details. The function BCCGo is identical to BCCG but with log link for mu.

## Usage

```
BCCG(mu.link = "identity", sigma.link = "log", nu.link = "identity")
BCCGo(mu.link = "log", sigma.link = "log", nu.link = "identity")
BCCGuntr(mu.link = "identity", sigma.link = "log", nu.link = "identity")
dBCCG(x, mu = 1, sigma = 0.1, nu = 1, log = FALSE)
pBCCG(q, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qBCCG(p, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
rBCCG(n, mu = 1, sigma = 0.1, nu = 1)
dBCCGo(x, mu = 1, sigma = 0.1, nu = 1, log = FALSE)
pBCCGo(q, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qBCCGo(p, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
rBCCGo(n, mu = 1, sigma = 0.1, nu = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter, other links are "inverse", "log" and "own" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other links are "inverse", "identity" and "own" |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter, other links are "inverse", "log" and "own" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness parameter values |

| | |
|---|---|
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability distribution function of the untrucated Box-Cox Cole and Green distribution, BCCGuntr, is defined as

$$f(y|\mu,\sigma,\nu) = \frac{1}{\sqrt{2\pi}\sigma}\frac{y^{\nu-1}}{\mu^{\nu}}\exp(-\frac{z^2}{2})$$

where if $\nu \neq 0$ then $z = [(y/\mu)^{\nu} - 1]/(\nu\sigma)$ else $z = \log(y/\mu)/\sigma$, for $y > 0$, $\mu > 0$, $\sigma > 0$ and $\nu = (-\infty, +\infty)$.

The Box-Cox Cole and Green distribution, BCCG, adjusts the above density $f(y|\mu,\sigma,\nu)$ for the truncation resulting from the condition $y > 0$. See Rigby and Stasinopoulos (2003a, 2003b) or pp. 439-441 of Rigby et al. (2019) for details.

## Value

BCCG() returns a gamlss.family object which can be used to fit a Cole and Green distribution in the gamlss() function. dBCCG() gives the density, pBCCG() gives the distribution function, qBCCG() gives the quantile function, and rBCCG() generates random deviates.

## Warning

The BCCGuntr distribution may be unsuitable for some combinations of the parameters (mainly for large $\sigma$) where the integrating constant is less than 0.99. A warning will be given if this is the case. The BCCG distribution is suitable for all combinations of the distributional parameters within their range [i.e. $\mu > 0$, $\sigma > 0$, $\nu = (-\infty, +\infty)$]

## Note

$\mu$ is the median of the distribution $\sigma$ is approximately the coefficient of variation (for small values of $\sigma$), and $\nu$ controls the skewness.

The BCCG distribution is suitable for all combinations of the parameters within their ranges [i.e. $\mu > 0, \sigma > 0, \text{and} \nu = (-\infty, \infty)$ ]

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Kalliope Akantziliotou

## References

Cole, T. J. and Green, P. J. (1992) Smoothing reference centile curves: the LMS method and penalized likelihood, *Statist. Med.* **11**, 1305–1319

Rigby, R. A. and Stasinopoulos, D. M. (2004). Smooth centile curves for skew and kurtotic data modelled using the Box-Cox Power Exponential distribution. *Statistics in Medicine*, **23**: 3053-3076.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R.A. Stasinopoulos, D.M. (2006). Using the Box-Cox *t* distribution in GAMLSS to mode skewnees and and kurtosis. *Statistical Modelling*, 6(3) :209. doi:10.1191/1471082X06st122oa

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/9780429298547 An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCPE, BCT

## Examples

```
BCCG()   # gives information about the default links for the Cole and Green distribution
# library(gamlss)
#data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=BCCG, data=abdom)
#plot(h)
plot(function(x) dBCCG(x, mu=5,sigma=.5,nu=-1), 0.0, 20,
 main = "The BCCG  density mu=5,sigma=.5,nu=-1")
plot(function(x) pBCCG(x, mu=5,sigma=.5,nu=-1), 0.0, 20,
 main = "The BCCG  cdf mu=5, sigma=.5, nu=-1")
```

---

BCPE *Box-Cox Power Exponential distribution for fitting a GAMLSS*

---

## Description

This function defines the Box-Cox Power Exponential distribution, a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss().

The functions dBCPE, pBCPE, qBCPE and rBCPE define the density, distribution function, quantile function and random generation for the Box-Cox Power Exponential distribution.

The function checkBCPE (very old) can be used, typically when a BCPE model is fitted, to check whether there exit a turning point of the distribution close to zero. It give the number of values of

the response below their minimum turning point and also the maximum probability of the lower tail below minimum turning point.

[The function `Biventer()` is the original version of the function suitable only for the untruncated BCPE distribution.] See Rigby and Stasinopoulos (2003) for details.

The function `BCPEo` is identical to `BCPE` but with log link for mu.

## Usage

```
BCPE(mu.link = "identity", sigma.link = "log", nu.link = "identity",
          tau.link = "log")
BCPEo(mu.link = "log", sigma.link = "log", nu.link = "identity",
          tau.link = "log")
BCPEuntr(mu.link = "identity", sigma.link = "log", nu.link = "identity",
          tau.link = "log")
dBCPE(x, mu = 5, sigma = 0.1, nu = 1, tau = 2, log = FALSE)
pBCPE(q, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE, log.p = FALSE)
qBCPE(p, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE, log.p = FALSE)
rBCPE(n, mu = 5, sigma = 0.1, nu = 1, tau = 2)
dBCPEo(x, mu = 5, sigma = 0.1, nu = 1, tau = 2, log = FALSE)
pBCPEo(q, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE,
       log.p = FALSE)
qBCPEo(p, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE,
       log.p = FALSE)
rBCPEo(n, mu = 5, sigma = 0.1, nu = 1, tau = 2)
checkBCPE(obj = NULL, mu = 10, sigma = 0.1, nu = 0.5, tau = 2,...)
```

## Arguments

| | |
|---|---|
| `mu.link` | Defines the `mu.link`, with "identity" link as the default for the `mu` parameter. Other links are "inverse", "log" and "own" |
| `sigma.link` | Defines the `sigma.link`, with "log" link as the default for the `sigma` parameter. Other links are "inverse", "identity" and "own" |
| `nu.link` | Defines the `nu.link`, with "identity" link as the default for the `nu` parameter. Other links are "inverse", "log" and "own" |
| `tau.link` | Defines the `tau.link`, with "log" link as the default for the `tau` parameter. Other links are "logshifted", "identity" and "own" |
| `x`, `q` | vector of quantiles |
| `mu` | vector of location parameter values |
| `sigma` | vector of scale parameter values |
| `nu` | vector of `nu` parameter values |
| `tau` | vector of `tau` parameter values |
| `log`, `log.p` | logical; if TRUE, probabilities p are given as log(p). |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| `p` | vector of probabilities. |

| | |
|---|---|
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| obj | a gamlss BCPE family object |
| ... | for extra arguments |

## Details

The probability density function of the untrucated Box Cox Power Exponential distribution, (BCPE.untr), is defined as

$$f(y|\mu,\sigma,\nu,\tau) = \frac{y^{\nu-1}\tau \exp[-\frac{1}{2}|\frac{z}{c}|^{\tau}]}{\mu^{\nu}\sigma c 2^{(1+1/\tau)}\Gamma(\frac{1}{\tau})}$$

where $c = [2^{(-2/\tau)}\Gamma(1/\tau)/\Gamma(3/\tau)]^{0.5}$, where if $\nu \neq 0$ then $z = [(y/\mu)^{\nu} - 1]/(\nu\sigma)$ else $z = \log(y/\mu)/\sigma$, for $y > 0$, $\mu > 0$, $\sigma > 0$, $\nu = (-\infty, +\infty)$ and $\tau > 0$ see pp. 450-451 of Rigby et al. (2019).

The Box-Cox Power Exponential, BCPE, adjusts the above density $f(y|\mu,\sigma,\nu,\tau)$ for the truncation resulting from the condition $y > 0$. See Rigby and Stasinopoulos (2003) for details.

## Value

BCPE() returns a gamlss.family object which can be used to fit a Box Cox Power Exponential distribution in the gamlss() function. dBCPE() gives the density, pBCPE() gives the distribution function, qBCPE() gives the quantile function, and rBCPE() generates random deviates.

## Warning

The BCPE.untr distribution may be unsuitable for some combinations of the parameters (mainly for large $\sigma$) where the integrating constant is less than 0.99. A warning will be given if this is the case.

The BCPE distribution is suitable for all combinations of the parameters within their ranges [i.e. $\mu > 0, \sigma > 0, \nu = (-\infty, \infty) \text{and} \tau > 0$ ]

## Note

$\mu$, is the median of the distribution, $\sigma$ is approximately the coefficient of variation (for small $\sigma$ and moderate nu>0), $\nu$ controls the skewness and $\tau$ the kurtosis of the distribution

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A. and Stasinopoulos, D. M. (2004). Smooth centile curves for skew and kurtotic data modelled using the Box-Cox Power Exponential distribution. *Statistics in Medicine*, **23**: 3053-3076.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCT

## Examples

```
# BCPE()   #
# library(gamlss)
# data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=BCPE, data=abdom)
#plot(h)
plot(function(x)dBCPE(x, mu=5,sigma=.5,nu=1, tau=3), 0.0, 15,
 main = "The BCPE  density mu=5,sigma=.5,nu=1, tau=3")
plot(function(x) pBCPE(x, mu=5,sigma=.5,nu=1, tau=3), 0.0, 15,
 main = "The BCPE  cdf mu=5, sigma=.5, nu=1, tau=3")
```

---

BCT *Box-Cox t distribution for fitting a GAMLSS*

---

## Description

The function BCT() defines the Box-Cox t distribution, a four parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

The functions dBCT, pBCT, qBCT and rBCT define the density, distribution function, quantile function and random generation for the Box-Cox t distribution.

[The function BCTuntr() is the original version of the function suitable only for the untruncated BCT distribution]. See Rigby and Stasinopoulos (2003) for details.

The function BCTo is identical to BCT but with log link for mu.

## Usage

```
BCT(mu.link = "identity", sigma.link = "log", nu.link = "identity",
          tau.link = "log")
BCTo(mu.link = "log", sigma.link = "log", nu.link = "identity",
          tau.link = "log")
BCTuntr(mu.link = "identity", sigma.link = "log", nu.link = "identity",
          tau.link = "log")
```

```
dBCT(x, mu = 5, sigma = 0.1, nu = 1, tau = 2, log = FALSE)
pBCT(q, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE, log.p = FALSE)
qBCT(p, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE, log.p = FALSE)
rBCT(n, mu = 5, sigma = 0.1, nu = 1, tau = 2)
dBCTo(x, mu = 5, sigma = 0.1, nu = 1, tau = 2, log = FALSE)
pBCTo(q, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE, log.p = FALSE)
qBCTo(p, mu = 5, sigma = 0.1, nu = 1, tau = 2, lower.tail = TRUE, log.p = FALSE)
rBCTo(n, mu = 5, sigma = 0.1, nu = 1, tau = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse", "log" and "own" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse","identity", "own" |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "inverse", "log", "own" |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "inverse", "identity" and "own" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of nu parameter values |
| tau | vector of tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the untruncated Box-Cox t distribution, BCTuntr, is given by

$$f(y|\mu,\sigma,\nu,\tau) = \frac{y^{\nu-1}}{\mu^{\nu}\sigma}\frac{\Gamma[(\tau+1)/2]}{\Gamma(1/2)\Gamma(\tau/2)\tau^{0.5}}[1+(1/\tau)z^2]^{-(\tau+1)/2}$$

where if $\nu \neq 0$ then $z = [(y/\mu)^{\nu}-1]/(\nu\sigma)$ else $z = \log(y/\mu)/\sigma$, for $y > 0$, $\mu > 0$, $\sigma > 0$, $\nu = (-\infty, +\infty)$ and $\tau > 0$ see pp. 450-451 of Rigby et al. (2019).

The Box-Cox *t* distribution, BCT, adjusts the above density $f(y|\mu,\sigma,\nu,\tau)$ for the truncation resulting from the condition $y > 0$. See Rigby and Stasinopoulos (2003) for details.

## Value

BCT() returns a gamlss.family object which can be used to fit a Box Cox-t distribution in the gamlss() function. dBCT() gives the density, pBCT() gives the distribution function, qBCT() gives the quantile function, and rBCT() generates random deviates.

## Warning

The use BCTuntr distribution may be unsuitable for some combinations of the parameters (mainly for large $\sigma$) where the integrating constant is less than 0.99. A warning will be given if this is the case.

The BCT distribution is suitable for all combinations of the parameters within their ranges [i.e. $\mu > 0, \sigma > 0, \nu = (-\infty, \infty) \text{and} \tau > 0$ ]

## Note

$\mu$ is the median of the distribution, $\sigma(\frac{\tau}{\tau-2})^{0.5}$ is approximate the coefficient of variation (for small $\sigma$ and moderate nu>0 and moderate or large $\tau$), $\nu$ controls the skewness and $\tau$ the kurtosis of the distribution

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R.A. Stasinopoulos, D.M. (2006). Using the Box-Cox *t* distribution in GAMLSS to mode skewnees and and kurtosis. *Statistical Modelling* 6(3):200. doi:10.1191/1471082X06st122oa.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCPE, BCCG

## Examples

```
BCT()   # gives information about the default links for the Box Cox t distribution
# library(gamlss)
#data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=BCT, data=abdom) #
#plot(h)
plot(function(x)dBCT(x, mu=5,sigma=.5,nu=1, tau=2), 0.0, 20,
 main = "The BCT  density mu=5,sigma=.5,nu=1, tau=2")
plot(function(x) pBCT(x, mu=5,sigma=.5,nu=1, tau=2), 0.0, 20,
 main = "The BCT  cdf mu=5, sigma=.5, nu=1, tau=2")
```

BE *The beta distribution for fitting a GAMLSS*

## Description

The functions `BE()` and `BEo()` define the beta distribution, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`. `BE()` has mean equal to the parameter `mu` and `sigma` as scale parameter, see below. `BEo()` is the original parameterizations of the beta distribution as in `dbeta()` with `shape1`=mu and `shape2`=sigma. The functions `dBE` and `dBEo`, `pBE` and `pBEo`, `qBE` and `qBEo` and finally `rBE` and `rBE` define the density, distribution function, quantile function and random generation for the `BE` and `BEo` parameterizations respectively of the beta distribution.

## Usage

```
BE(mu.link = "logit", sigma.link = "logit")
dBE(x, mu = 0.5, sigma = 0.2, log = FALSE)
pBE(q, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
qBE(p, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
rBE(n, mu = 0.5, sigma = 0.2)
BEo(mu.link = "log", sigma.link = "log")
dBEo(x, mu = 0.5, sigma = 0.2, log = FALSE)
pBEo(q, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
qBEo(p, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
```

## Arguments

| | |
|---|---|
| `mu.link` | the `mu` link function with default `logit` |
| `sigma.link` | the `sigma` link function with default `logit` |
| `x`, `q` | vector of quantiles |
| `mu` | vector of location parameter values |
| `sigma` | vector of scale parameter values |
| `log`, `log.p` | logical; if TRUE, probabilities p are given as log(p). |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| `p` | vector of probabilities. |
| `n` | number of observations. If `length(n) > 1`, the length is taken to be the number required |

## Details

The standard parametrization of the beta distribution is given as:

$$f(y|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} y^{\alpha-1}(1-y)^{\beta-1}$$

for $y = (0,1)$, $\alpha > 0$ and $\beta > 0$.

The first gamlss implementation the beta distribution is called BEo, and it is identical to the standard parametrization with $\alpha = \mu$ and $\beta = \sigma$, see pp. 460-461 of Rigby et al. (2019):

$$f(y|\mu,\sigma) = \frac{1}{B(\mu,\sigma)} y^{\mu-1}(1-y)^{\sigma-1}$$

for $y = (0,1)$, $\mu > 0$ and $\sigma > 0$. The problem with this parametrization is that with mean $E(y) = \mu/(\mu+\sigma)$ it is not convenient for modelling the response y as function of the explanatory variables. The second parametrization, BE see pp. 461-463 of Rigby et al. (2019), is using

$$\mu = \frac{\alpha}{\alpha+\beta}$$

$$\sigma = \frac{1}{\alpha+\beta+1)^{1/2}}$$

(of the standard parametrization) and it is more convenient because $\mu$ now is the mean with variance equal to $Var(y) = \sigma^2\mu(1-\mu)$. Note however that $0 < \mu < 1$ and $0 < \sigma < 1$.

## Value

BE() and BEo() return a gamlss.family object which can be used to fit a beta distribution in the gamlss() function.

## Note

Note that for BE, mu is the mean and sigma a scale parameter contributing to the variance of y

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BE, LOGITNO, GB1, BEINF

## Examples

```
BE()# gives information about the default links for the beta distribution
dat1<-rBE(100, mu=.3, sigma=.5)
hist(dat1)
#library(gamlss)
# mod1<-gamlss(dat1~1,family=BE) # fits a constant for mu and sigma
#fitted(mod1)[1]
#fitted(mod1,"sigma")[1]
plot(function(y) dBE(y, mu=.1 ,sigma=.5), 0.001, .999)
plot(function(y) pBE(y, mu=.1 ,sigma=.5), 0.001, 0.999)
plot(function(y) qBE(y, mu=.1 ,sigma=.5), 0.001, 0.999)
plot(function(y) qBE(y, mu=.1 ,sigma=.5, lower.tail=FALSE), 0.001, .999)
dat2<-rBEo(100, mu=1, sigma=2)
#mod2<-gamlss(dat2~1,family=BEo) # fits a constant for mu and sigma
#fitted(mod2)[1]
#fitted(mod2,"sigma")[1]
```

---

# BEINF *The beta inflated distribution for fitting a GAMLSS*

---

## Description

The function BEINF() defines the beta inflated distribution, a four parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The beta inflated is similar to the beta but allows zeros and ones as values for the response variable. The two extra parameters model the probabilities at zero and one.

The functions BEINF0() and BEINF1() are three parameter beta inflated distributions allowing zeros or ones only at the response respectively. BEINF0() and BEINF1() are re-parameterize versions of the distributions BEZI and BEOI contributed to gamlss by Raydonal Ospina (see Ospina and Ferrari (2010)).

The functions dBEINF, pBEINF, qBEINF and rBEINF define the density, distribution function, quantile function and random generation for the BEINF parametrization of the beta inflated distribution.

The functions dBEINF0, pBEINF0, qBEINF0 and rBEINF0 define the density, distribution function, quantile function and random generation for the BEINF0 parametrization of the beta inflated at zero distribution.

The functions dBEINF1, pBEINF1, qBEINF1 and rBEINF1 define the density, distribution function, quantile function and random generation for the BEINF1 parametrization of the beta inflated at one distribution.

plotBEINF, plotBEINF0 and plotBEINF1 can be used to plot the distributions. meanBEINF, meanBEINF0 and meanBEINF1 calculates the expected value of the response for a fitted model.

## Usage

```
BEINF(mu.link = "logit", sigma.link = "logit", nu.link = "log",
      tau.link = "log")
BEINF0(mu.link = "logit", sigma.link = "logit", nu.link = "log")
BEINF1(mu.link = "logit", sigma.link = "logit", nu.link = "log")

dBEINF(x, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
       log = FALSE)
dBEINF0(x, mu = 0.5, sigma = 0.1, nu = 0.1, log = FALSE)
dBEINF1(x, mu = 0.5, sigma = 0.1, nu = 0.1, log = FALSE)

pBEINF(q, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
       lower.tail = TRUE, log.p = FALSE)
pBEINF0(q, mu = 0.5, sigma = 0.1, nu = 0.1,
       lower.tail = TRUE, log.p = FALSE)
pBEINF1(q, mu = 0.5, sigma = 0.1, nu = 0.1,
        lower.tail = TRUE, log.p = FALSE)

qBEINF(p, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
       lower.tail = TRUE, log.p = FALSE)
qBEINF0(p, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
        lower.tail = TRUE, log.p = FALSE)
qBEINF1(p, mu = 0.5, sigma = 0.1, nu = 0.1,
        lower.tail = TRUE, log.p = FALSE)

rBEINF(n, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1)
rBEINF0(n, mu = 0.5, sigma = 0.1, nu = 0.1)
rBEINF1(n, mu = 0.5, sigma = 0.1, nu = 0.1)

plotBEINF(mu = 0.5, sigma = 0.5, nu = 0.5, tau = 0.5,
          from = 0.001, to = 0.999, n = 101, ...)
plotBEINF0(mu = 0.5, sigma = 0.5, nu = 0.5,
          from = 1e-04, to = 0.9999, n = 101, ...)
plotBEINF1(mu = 0.5, sigma = 0.5, nu = 0.5,
          from = 1e-04, to = 0.9999, n = 101, ...)

meanBEINF(obj)
meanBEINF0(obj)
meanBEINF1(obj)
```

## Arguments

| | |
|---|---|
| mu.link | the mu link function with default logit |
| sigma.link | the sigma link function with default logit |
| nu.link | the nu link function with default log |
| tau.link | the tau link function with default log |

| | |
|---|---|
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of parameter values modelling the probability at zero |
| tau | vector of parameter values modelling the probability at one |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| from | where to start plotting the distribution from |
| to | up to where to plot the distribution |
| obj | a fitted BEINF object |
| ... | other graphical parameters for plotting |

## Details

The beta inflated distribution is defined as

$$f(y) = p_0 \quad \textit{if} \quad y = 0$$

$$f(y) = p_1 \quad \textit{if} \quad y = 1$$

$$f(y|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} y^{\alpha-1}(1-y)^{\beta-1} \quad \textit{otherwise}$$

for $0 \leq y \leq 1$, $\alpha > 0$, $\beta > 0$, $0 < p_0 < 1$, $0 < p_1 < 1$ and $0 < p_0 + p_1 < 1$.

The GAMLSS function BEINF() re-parametrize the parameters as

$$\mu = \frac{\alpha}{\alpha+\beta}$$

$$\sigma = \frac{1}{\alpha+\beta+1}$$

$$\nu = \frac{p_0}{p_2}$$

$$\tau = \frac{p_1}{p_2}$$

where $p_2 = 1 - p_0 - p_1$ so $0 < \mu < 1$, $0 < \sigma < 1$, $\nu > 0$ and $\tau > 0$ see pp. 466-467 of Rigby et al. (2019).

The beta inflated at zero distribution is defined as

$$f(y) = p_0 \quad \textit{if} \quad y = 0$$

$$f(y|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} y^{\alpha-1}(1-y)^{\beta-1} \quad \textit{otherwise}$$

for $0 \leq y < 1$, $\alpha > 0$, $\beta > 0$ and $0 < p_0 < 1$.

The GAMLSS function `BEINF0()` re-parametrize the parameters as

$$\mu = \frac{\alpha}{\alpha + \beta}$$

$$\sigma = \frac{1}{\alpha + \beta + 1}$$

$$\nu = \frac{p_0}{1 - P_0}$$

so $0 < \mu < 1$, $0 < \sigma < 1$ and $\nu > 0$ see pp. 467-468 of Rigby et al. (2019).

The beta inflated at 1 distribution is defined as

$$f(y|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} y^{\alpha - 1}(1 - y)^{\beta - 1} \quad \textit{if} \quad 0 < y < 1$$

$$f(y) = p_1 \qquad \textit{if} \quad y = 1$$

for $0 < y \leq 1$, $\alpha > 0$, $\beta > 0$ and $0 < p_1 < 1$.

The GAMLSS function `BEINF1()` re-parametrize the parameters as

$$\mu = \frac{\alpha}{\alpha + \beta}$$

$$\sigma = \frac{1}{\alpha + \beta + 1}$$

$$\nu = \frac{p_1}{1 - P_1}$$

so $0 < \mu < 1$, $0 < \sigma < 1$ and $\nu > 0$ see pp. 468-469 of Rigby et al. (2019).

## Value

returns a `gamlss.family` object which can be used to fit a beta inflated distribution in the `gamlss()` function. ...

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Ospina R. and Ferrari S. L. P. (2010) Inflated beta distributions, *Statistical Papers*, **23**, 111-126.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC,doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BE, BEo, BEZI, BEOI

## Examples

```
BEINF()# gives information about the default links for the beta inflated distribution
BEINF0()
BEINF1()
# plotting the distributions
op<-par(mfrow=c(2,2))
plotBEINF( mu =.5 , sigma=.5, nu = 0.5, tau = 0.5, from = 0, to=1, n = 101)
plotBEINF0( mu =.5 , sigma=.5, nu = 0.5,  from = 0, to=1, n = 101)
plotBEINF1( mu =.5 , sigma=.5, nu = 0.5,  from = 0.001, to=1, n = 101)
curve(dBE(x, mu =.5, sigma=.5),  0.01, 0.999)
par(op)
# plotting the cdf
op<-par(mfrow=c(2,2))
plotBEINF( mu =.5 , sigma=.5, nu = 0.5, tau = 0.5, from = 0, to=1, n = 101, main="BEINF")
plotBEINF0( mu =.5 , sigma=.5, nu = 0.5,  from = 0, to=1, n = 101, main="BEINF0")
plotBEINF1( mu =.5 , sigma=.5, nu = 0.5,  from = 0.001, to=1, n = 101, main="BEINF1")
curve(dBE(x, mu =.5, sigma=.5),  0.01, 0.999, main="BE")
par(op)
#-------------------------------------------
op<-par(mfrow=c(2,2))
plotBEINF( mu =.5 , sigma=.5, nu = 0.5, tau = 0.5, from = 0, to=1, n = 101, main="BEINF")
plotBEINF0( mu =.5 , sigma=.5, nu = 0.5,  from = 0, to=1, n = 101, main="BEINF0")
plotBEINF1( mu =.5 , sigma=.5, nu = 0.5,  from = 0.001, to=1, n = 101, main="BEINF1")
curve(dBE(x, mu =.5, sigma=.5),  0.01, 0.999, main="BE")
par(op)
#-------------------------------------------
op<-par(mfrow=c(2,2))
curve( pBEINF(x, mu=.5 ,sigma=.5, nu = 0.5, tau = 0.5,), 0, 1, ylim=c(0,1), main="BEINF" )
curve(pBEINF0(x, mu=.5 ,sigma=.5, nu = 0.5), 0, 1, ylim=c(0,1), main="BEINF0")
curve(pBEINF1(x, mu=.5 ,sigma=.5, nu = 0.5), 0, 1, ylim=c(0,1), main="BEINF1")
curve(    pBE(x, mu=.5 ,sigma=.5), .001, .99, ylim=c(0,1), main="BE")
par(op)
#-------------------------------------------
op<-par(mfrow=c(2,2))
curve(qBEINF(x, mu=.5 ,sigma=.5, nu = 0.5, tau = 0.5), .01, .99, main="BEINF" )
curve(qBEINF0(x, mu=.5 ,sigma=.5, nu = 0.5), .01, .99, main="BEINF0" )
curve(qBEINF1(x, mu=.5 ,sigma=.5, nu = 0.5), .01, .99, main="BEINF1" )
curve(qBE(x, mu=.5 ,sigma=.5), .01, .99 , main="BE")
par(op)

#-------------------------------------------
op<-par(mfrow=c(2,2))
hist(rBEINF(200, mu=.5 ,sigma=.5, nu = 0.5, tau = 0.5))
hist(rBEINF0(200, mu=.5 ,sigma=.5, nu = 0.5))
hist(rBEINF1(200, mu=.5 ,sigma=.5, nu = 0.5))
hist(rBE(200, mu=.5 ,sigma=.5))
par(op)
# fit a model to the data
```

```
# library(gamlss)
#m1<-gamlss(dat~1,family=BEINF)
#meanBEINF(m1)[1]
```

---

BEOI *The one-inflated beta distribution for fitting a GAMLSS*

---

## Description

The function BEOI() defines the one-inflated beta distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The one-inflated beta is similar to the beta distribution but allows ones as y values. This distribution is an extension of the beta distribution using a parameterization of the beta law that is indexed by mean and precision parameters (Ferrari and Cribari-Neto, 2004). The extra parameter models the probability at one. The functions dBEOI, pBEOI, qBEOI and rBEOI define the density, distribution function, quantile function and random generation for the BEOI parameterization of the one-inflated beta distribution. plotBEOI can be used to plot the distribution. meanBEOI calculates the expected value of the response for a fitted model.

## Usage

```
BEOI(mu.link = "logit", sigma.link = "log", nu.link = "logit")

dBEOI(x, mu = 0.5, sigma = 1, nu = 0.1, log = FALSE)

pBEOI(q, mu = 0.5, sigma = 1, nu = 0.1, lower.tail = TRUE, log.p = FALSE)

qBEOI(p, mu = 0.5, sigma = 1, nu = 0.1, lower.tail = TRUE,
        log.p = FALSE)

rBEOI(n, mu = 0.5, sigma = 1, nu = 0.1)

plotBEOI(mu = .5, sigma = 1, nu = 0.1, from = 0.001, to = 1, n = 101,
    ...)

meanBEOI(obj)
```

## Arguments

| | |
|---|---|
| mu.link | the mu link function with default logit |
| sigma.link | the sigma link function with default log |
| nu.link | the nu link function with default logit |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of precision parameter values |
| nu | vector of parameter values modelling the probability at one |

| | |
|---|---|
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| from | where to start plotting the distribution from |
| to | up to where to plot the distribution |
| obj | a fitted BEOI object |
| ... | other graphical parameters for plotting |

## Details

The one-inflated beta distribution is given as

$$f(y) = \nu \quad \textit{if} \quad y = 1$$

$$f(y|\mu,\sigma) = (1-\nu)\frac{\Gamma(\sigma)}{\Gamma(\mu\sigma)\Gamma((1-\mu)\sigma)} y^{\mu\sigma}(1-y)^{((1-\mu)\sigma)-1} \quad \textit{otherwise}$$

The parameters satisfy $0 < \mu < 0$, $\sigma > 0$ and $0 < \nu < 1$.

Here $E(y) = \nu + (1-\nu)\mu$ and $Var(y) = (1-\nu)\frac{\mu(1-\mu)}{\sigma+1} + \nu(1-\nu)(1-\mu)^2$.

## Value

returns a gamlss.family object which can be used to fit a one-inflated beta distribution in the gamlss() function.

## Note

This work is part of my PhD project at the University of Sao Paulo under the supervion of Professor Silvia Ferrari. My thesis is concerned with regression modelling of rates and proportions with excess of zeros and/or ones

## Author(s)

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## References

Ferrari, S.L.P., Cribari-Neto, F. (2004). Beta regression for modelling rates and proportions. *Journal of Applied Statistics*, **31** (1), 799-815.

Ospina R. and Ferrari S. L. P. (2010) Inflated beta distributions, *Statistical Papers*, **23**, 111-126.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape (with discussion). *Applied Statistics*, **54** (3), 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BEOI

## Examples

```
BEOI()# gives information about the default links for the BEOI distribution
# plotting the distribution
plotBEOI( mu =0.5 , sigma=5, nu = 0.1, from = 0.001, to=1, n = 101)
# plotting the cdf
plot(function(y) pBEOI(y, mu=.5 ,sigma=5, nu=0.1), 0.001, 0.999)
# plotting the inverse cdf
plot(function(y) qBEOI(y, mu=.5 ,sigma=5, nu=0.1), 0.001, 0.999)
# generate random numbers
dat<-rBEOI(100, mu=.5, sigma=5, nu=0.1)
# fit a model to the data.
# library(gamlss)
#mod1<-gamlss(dat~1,sigma.formula=~1, nu.formula=~1, family=BEOI)
#fitted(mod1)[1]
#summary(mod1)
#fitted(mod1,"mu")[1]        #fitted mu
#fitted(mod1,"sigma")[1]     #fitted sigma
#fitted(mod1,"nu")[1]        #fitted nu
#meanBEOI(mod1)[1] # expected value of the response
```

---

BEZI *The zero-inflated beta distribution for fitting a GAMLSS*

---

## Description

The function BEZI() defines the zero-inflated beta distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The zero-inflated beta is similar to the beta distribution but allows zeros as y values. This distribution is an extension of the beta distribution using a parameterization of the beta law that is indexed by mean and precision parameters (Ferrari and Cribari-Neto, 2004). The extra parameter models the probability at zero. The functions dBEZI, pBEZI, qBEZI and rBEZI define the density, distribution function, quantile function and random generation for the BEZI parameterization of the zero-inflated beta distribution. plotBEZI can be used to plot the distribution. meanBEZI calculates the expected value of the response for a fitted model.

## Usage

```
BEZI(mu.link = "logit", sigma.link = "log", nu.link = "logit")

dBEZI(x, mu = 0.5, sigma = 1, nu = 0.1, log = FALSE)

pBEZI(q, mu = 0.5, sigma = 1, nu = 0.1, lower.tail = TRUE, log.p = FALSE)

qBEZI(p, mu = 0.5, sigma = 1, nu = 0.1, lower.tail = TRUE,
        log.p = FALSE)

rBEZI(n, mu = 0.5, sigma = 1, nu = 0.1)

plotBEZI(mu = .5, sigma = 1, nu = 0.1, from = 0, to = 0.999, n = 101,
    ...)

meanBEZI(obj)
```

## Arguments

| | |
|---|---|
| mu.link | the mu link function with default logit |
| sigma.link | the sigma link function with default log |
| nu.link | the nu link function with default logit |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of precision parameter values |
| nu | vector of parameter values modelling the probability at zero |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| from | where to start plotting the distribution from |
| to | up to where to plot the distribution |
| obj | a fitted BEZI object |
| ... | other graphical parameters for plotting |

## Details

The zero-inflated beta distribution is given as

$$f(y) = \nu$$

if $(y = 0)$

$$f(y|\mu, \sigma) = (1 - \nu)\frac{\Gamma(\sigma)}{\Gamma(\mu\sigma)\Gamma((1 - \mu)\sigma)} y^{\mu\sigma}(1 - y)^{((1-\mu)\sigma)-1}$$

if $y = (0, 1)$. The parameters satisfy $0 < \mu < 0$, $\sigma > 0$ and $0 < \nu < 1$.

Here $E(y) = (1 - \nu)\mu$ and $Var(y) = (1 - \nu)\frac{\mu(1-\mu)}{\sigma+1} + \nu(1 - \nu)\mu^2$.

## Value

returns a `gamlss.family` object which can be used to fit a zero-inflated beta distribution in the `gamlss()` function.

## Note

This work is part of my PhD project at the University of Sao Paulo under the supervion of Professor Silvia Ferrari. My thesis is concerned with regression modelling of rates and proportions with excess of zeros and/or ones

## Author(s)

Raydonal Ospina, Department of Statistics, University of Sao Paulo, Brazil.

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## References

Ferrari, S.L.P., Cribari-Neto, F. (2004). Beta regression for modelling rates and proportions. *Journal of Applied Statistics*, **31** (1), 799-815.

Ospina R. and Ferrari S. L. P. (2010) Inflated beta distributions, *Statistical Papers*, **23**, 111-126.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape (with discussion). *Applied Statistics*, **54** (3), 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BEZI

## Examples

```
 BEZI()# gives information about the default links for the BEZI distribution
# plotting the distribution
plotBEZI( mu =0.5 , sigma=5, nu = 0.1, from = 0, to=0.99, n = 101)
# plotting the cdf
plot(function(y) pBEZI(y, mu=.5 ,sigma=5, nu=0.1), 0, 0.999)
# plotting the inverse cdf
plot(function(y) qBEZI(y, mu=.5 ,sigma=5, nu=0.1), 0, 0.999)
# generate random numbers
dat<-rBEZI(100, mu=.5, sigma=5, nu=0.1)
# fit a model to the data. Tits a constant for mu, sigma and nu
# library(gamlss)
```

```
#mod1<-gamlss(dat~1,sigma.formula=~1, nu.formula=~1, family=BEZI)
#fitted(mod1)[1]
#summary(mod1)
#fitted(mod1,"mu")[1]         #fitted mu
#fitted(mod1,"sigma")[1]      #fitted sigma
#fitted(mod1,"nu")[1]         #fitted nu
#meanBEZI(mod1)[1] # expected value of the response
```

---

BI *Binomial distribution for fitting a GAMLSS*

---

## Description

The `BI()` function defines the binomial distribution, a one parameter family distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`. The functions `dBI`, `pBI`, `qBI` and `rBI` define the density, distribution function, quantile function and random generation for the binomial, `BI()`, distribution.

## Usage

```
BI(mu.link = "logit")
dBI(x, bd = 1, mu = 0.5, log = FALSE)
pBI(q, bd = 1, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
qBI(p, bd = 1, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
rBI(n, bd = 1, mu = 0.5)
```

## Arguments

| | |
|---|---|
| `mu.link` | Defines the `mu.link`, with "logit" link as the default for the `mu` parameter. Other links are "probit" and "cloglog"'(complementary log-log) |
| `x` | vector of (non-negative integer) quantiles |
| `mu` | vector of positive probabilities |
| `bd` | vector of binomial denominators |
| `p` | vector of probabilities |
| `q` | vector of quantiles |
| `n` | number of random values to return |
| `log, log.p` | logical; if TRUE, probabilities p are given as log(p) |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

Definition file for binomial distribution.

$$f(y|\mu) = \frac{\Gamma(n+1)}{\Gamma(y+1)\Gamma(n-y+1)}\mu^y(1-\mu)^{(n-y)}$$

for $y = 0, 1, 2, ..., n$ and $0 < \mu < 1$ see pp. 521-522 of Rigby *et al.* (2019).

## Value

returns a gamlss.family object which can be used to fit a binomial distribution in the gamlss() function.

## Note

The response variable should be a matrix containing two columns, the first with the count of successes and the second with the count of failures. The parameter mu represents a probability parameter with limits $0 < \mu < 1$. $n\mu$ is the mean of the distribution where n is the binomial denominator.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, ZABI, ZIBI

## Examples

```
 BI()# gives information about the default links for the Binomial distribution
# data(aep)
# library(gamlss)
# h<-gamlss(y~ward+loglos+year, family=BI, data=aep)
# plot of the binomial distribution
curve(dBI(x, mu = .5, bd=10), from=0, to=10, n=10+1, type="h")
tN <- table(Ni <- rBI(1000, mu=.2, bd=10))
r <- barplot(tN, col='lightblue')
```

BNB *Beta Negative Binomial distribution for fitting a GAMLSS*

## Description

The BNB() function defines the beta negative binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

The functions dBNB, pBNB, qBNB and rBNB define the density, distribution function, quantile function and random generation for the beta negative binomial distribution, BNB().

The functions ZABNB() and ZIBNB() are the zero adjusted (hurdle) and zero inflated versions of the beta negative binomial distribution, respectively. That is four parameter distributions.

The functions dZABNB, dZIBNB, pZABNB,pZIBNB, qZABNB qZIBNB rZABNB and rZIBNB define the probability, cumulative, quantile and random generation functions for the zero adjusted and zero inflated beta negative binomial distributions, ZABNB(), ZIBNB(), respectively.

## Usage

```
BNB(mu.link = "log", sigma.link = "log", nu.link = "log")
dBNB(x, mu = 1, sigma = 1, nu = 1, log = FALSE)
pBNB(q, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qBNB(p, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE,
     max.value = 10000)
rBNB(n, mu = 1, sigma = 1, nu = 1, max.value = 10000)

ZABNB(mu.link = "log", sigma.link = "log", nu.link = "log",
      tau.link = "logit")
dZABNB(x, mu = 1, sigma = 1, nu = 1, tau = 0.1, log = FALSE)
pZABNB(q, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE)
qZABNB(p, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE, max.value = 10000)
rZABNB(n, mu = 1, sigma = 1, nu = 1, tau = 0.1, max.value = 10000)

ZIBNB(mu.link = "log", sigma.link = "log", nu.link = "log",
      tau.link = "logit")
dZIBNB(x, mu = 1, sigma = 1, nu = 1, tau = 0.1, log = FALSE)
pZIBNB(q, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE)
qZIBNB(p, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE, max.value = 10000)
rZIBNB(n, mu = 1, sigma = 1, nu = 1, tau = 0.1, max.value = 10000)
```

## Arguments

| | |
|---|---|
| mu.link | The link function for mu |
| sigma.link | The link function for sigma |

| | |
|---|---|
| nu.link | The link function for nu |
| tau.link | The link function for tau |
| x | vector of (non-negative integer) |
| mu | vector of positive means |
| sigma | vector of positive dispersion parameter |
| nu | vector of a positive parameter |
| tau | vector of probabilities |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| max.value | a constant, set to the default value of 10000 for how far the algorithm should look for q |

## Details

The probability function of the BNB is

$$P(Y=y|\mu,\sigma,\nu)=\frac{\Gamma(y+\nu^{-1})B(y+\mu\sigma^{-1}\nu,\sigma^{-1}+\nu^{-1}+1)}{\Gamma(y+1)\Gamma(\nu^{-1})B(\mu\sigma^{-1}\nu,\sigma^{-1}+1)}$$

for $y=0,1,2,3,...$, $\mu>0$, $\sigma>0$ and $\nu>0$, see pp 502-503 of Rigby *et al.* (2019).

The distribution has mean $\mu$.

The definition of the zero adjusted beta negative binomial distribution, ZABNB and the the zero inflated beta negative binomial distribution, ZIBNB, are given in p. 517 and pp. 519 of of Rigby *et al.* (2019), respectively.

## Value

returns a gamlss.family object which can be used to fit a Poisson distribution in the gamlss() function.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

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(see also https://www.gamlss.com/).

## See Also

NBI, NBII

## Examples

```
BNB()   # gives information about the default links for the beta negative binomial
# plotting the distribution
plot(function(y) dBNB(y, mu = 10, sigma = 0.5, nu=2), from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rBNB(1000, mu=5, sigma=0.5, nu=2))
r <- barplot(tN, col='lightblue')

ZABNB()
ZIBNB()
# plotting the distribution
plot(function(y) dZABNB(y, mu = 10, sigma = 0.5, nu=2, tau=.1),
     from=0, to=40, n=40+1, type="h")
plot(function(y) dZIBNB(y, mu = 10, sigma = 0.5, nu=2, tau=.1),
     from=0, to=40, n=40+1, type="h")
## Not run:
library(gamlss)
data(species)
species <- transform(species, x=log(lake))
m6 <- gamlss(fish~ pb(x), sigma.fo=~1, data=species, family=BNB)

## End(Not run)
```

---

# checklink *Set the Right Link Function for Specified Parameter and Distribution*

---

## Description

This function is used within the distribution family specification of a GAMLSS model to define the right link for each of the parameters of the distribution. This function should not be called by the user unless he/she specify a new distribution family or wishes to change existing link functions in the parameters.

## Usage

```
checklink(which.link = NULL, which.dist = NULL, link = NULL, link.List = NULL)
```

## Arguments

| | |
|---|---|
| which.link | which parameter link e.g. which.link="mu.link" |
| which.dist | which distribution family e.g. which.dist="Cole.Green" |
| link | a repetition of which.link e.g. link=substitute(mu.link) |
| link.List | what link function are required e.g. link.List=c("inverse", "log", "identity") |

## Value

Defines the right link for each parameter

## Author(s)

Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC,doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

---

count_1_31 *A set of functions to plot gamlss.family distributions*

---

## Description

Those functions are used in the distribution book of gamlss, see Rigby et. al 2019.

## Usage

```
binom_1_31(family = BI, mu = c(0.1, 0.5, 0.7), bd = NULL, miny = 0,
           maxy = 20, cex.axis = 1.2, cex.all = 1.5)

binom_2_33(family = BB, mu = c(0.1, 0.5, 0.8), sigma = c(0.5, 1, 2),
           bd = NULL, miny = 0, maxy = 10, cex.y.axis = 1.5,
           cex.all = 1.5)

binom_3_33(family = ZIBB, mu = c(0.1, 0.5, 0.8), sigma = c(0.5, 1, 2),
           nu = c(0.01, 0.3), bd = NULL, miny = 0, maxy = 10,
           cex.axis = 1.5, cex.all = 1.5, cols = c("darkgray", "black"),
           spacing = 0.3, legend.cex=1, legend.x="topright",
          legend.where=c("left","right", "center"))

contR_2_12(family = "NO", mu = c(0, -1, 1), sigma = c(1, 0.5, 2),
          cols=c(gray(.1),gray(.2),gray(.3)),
          ltype = c(1, 2, 3), maxy = 7,
          no.points = 201, y.axis.lim = 1.1,
          cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topleft" )

contR_3_11(family = "PE", mu = 0, sigma = 1, nu = c(1, 2, 3),
          cols=c(gray(.1),gray(.2),gray(.3)), maxy = 7, no.points = 201,
          ltype = c(1, 2, 3), y.axis.lim = 1.1, cex.axis = 1.5,
          cex.all = 1.5, legend.cex=1, legend.x="topleft")

contR_4_13(family = "SEP3", mu = 0, sigma = 1, nu = c(0.5, 1, 2),
         tau = c(1, 2, 5), cols=c(gray(.1),gray(.2),gray(.3)), maxy = 7,
         no.points = 201, ltype = c(1, 2, 3),
         y.axis.lim = 1.1, cex.axis = 1.5, cex.all = 1.5,
         legend.cex=1, legend.x="topleft",
          legend.where=c("left","right"))

contRplus_1_1(family = EXP, mu = c(1, 2, 3),
       cols = c(gray(0.1), gray(0.2), gray(0.3)), maxy = 4,
       no.points = 201, y.axis.lim = 1.1, ltype = c(1, 2, 3),
       cex.axis = 1.5, cex.all = 1.5, legend.cex = 1,
       legend.x = "topright")

contRplus_2_11(family = GA, mu = 1, sigma = c(0.1, 0.6, 1),
          cols=c(gray(.1),gray(.2),gray(.3)),
          maxy = 4, no.points = 201,
          y.axis.lim = 1.1, ltype = c(1, 2, 3),
           cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topright")

contRplus_3_13(family = "BCCG", mu = 1, sigma = c(0.15, 0.2, 0.5),
          nu = c(-2, 0, 4),
```

```
          cols=c(gray(.1),gray(.2),gray(.3)),
          maxy = 4, ltype = c(1, 2, 3),
          no.points = 201, y.axis.lim = 1.1,
          cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topright",
          legend.where=c("left","right"))

contRplus_4_33(family = BCT, mu = 1, sigma = c(0.15, 0.2, 0.5),
          nu = c(-4, 0, 2), tau = c(100, 5, 1),
          cols=c(gray(.1),gray(.2),gray(.3)),
          maxy = 4, ltype = c(1, 2, 3),
          no.points = 201, y.axis.lim = 1.1,
          cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topright",
          legend.where=c("left","right"))

contR01_2_13(family = "BE", mu = c(0.2, 0.5, 0.8), sigma = c(0.2, 0.5, 0.8),
            cols=c(gray(.1),gray(.2),gray(.3)),
            ltype = c(1, 2, 3), maxy = 7, no.points = 201,
            y.axis.lim = 1.1, maxYlim = 10,
            cex.axis = 1.5, cex.all = 1.5,
            legend.cex=1, legend.x="topright",
            legend.where=c("left","right", "center"))

contR01_4_33(family = GB1, mu = c(0.5), sigma = c(0.2, 0.5, 0.7),
            nu = c(1, 2, 5), tau = c(0.5, 1, 2),
             cols=c(gray(.1),gray(.2),gray(.3)),
             maxy = 0.999, ltype = c(1, 2, 3),
             no.points = 201, y.axis.lim = 1.1,
             maxYlim = 10,cex.axis = 1.5, cex.all = 1.5,
              legend.cex=1, legend.x="topright",
              legend.where=c("left","right", "center"))

count_1_31(family = PO, mu = c(1, 2, 5), miny = 0, maxy = 10,
           cex.y.axis = 1.2, cex.all = 1.5)

count_1_22(family = PO, mu = c(1, 2, 5, 10), miny = 0,
           maxy = 20, cex.y.axis = 1.2, cex.all = 1.5)

count_2_32(family = NBI, mu = c(0.5, 1, 5), sigma = c(0.1, 2),
           miny = 0, maxy = 10, cex.y.axis = 1.5, cex.all = 1.5)

count_2_32R(family = NBI, mu = c(1, 2), sigma = c(0.1, 1, 2),
           miny = 0, maxy = 10, cex.y.axis = 1.5, cex.all = 1.5)

count_2_33(family = NBI, mu = c(0.1, 1, 2), sigma = c(0.5, 1, 2),
          miny = 0, maxy = 10, cex.y.axis = 1.5, cex.all = 1.5)
```

```
count_3_32(family = SICHEL, mu = c(1, 5, 10), sigma = c(0.5, 1),
           nu = c(-0.5, 0.5), miny = 0, maxy = 10, cex.axis = 1.5,
           cex.all = 1.5, cols = c("darkgray", "black"), spacing = 0.2,
           legend.cex=1, legend.x="topright",
                  legend.where=c("left","right"))

count_3_33(family = SICHEL, mu = c(1, 5, 10), sigma = c(0.5, 1, 2),
           nu = c(-0.5, 0.5, 1), miny = 0, maxy = 10, cex.y.axis = 1.5,
           cex.all = 1.5, cols = c("darkgray", "black"), spacing = 0.3,
           legend.cex=1, legend.x="topright",
           legend.where=c("left","right", "center"))
```

## Arguments

| | |
|---|---|
| family | a gamlss family distribution |
| mu | the mu parameter values |
| sigma | The sigma parameter values |
| nu | the nu parameter values |
| tau | the tau parameter values |
| bd | the binomial denominator |
| miny | minimal value for the y axis |
| maxy | maximal value for the y axis |
| cex.axis | the size of the letters in the two axes |
| cex.y.axis | the size of the letters in the y axe |
| cex.all | the overall size of all plotting characters |
| cols | colours |
| spacing | spacing between plots |
| ltype | The type of lines used |
| no.points | the number of points in the curve |
| y.axis.lim | the maximum value for the y axis |
| maxYlim | the maximum permissible value for Y |
| legend.cex | the size of the legend |
| legend.x | where in the figure to put the legend |
| legend.where | where in the whole plot to put the legend |

## Details

Th function plot different types of continuous and discrete distributions: i) contR: continuous distribution defined on minus infinity to plus infinity, ii) contRplus: continuous distribution defined from zero to plus infinity, iii) contR01: continuous distribution defined from zero to 1, iv) bimom binomial type discrete distributions, v) count count type discrete distributions.

The first number after the first underline in the name of the function indicates the number of parameters in the distribution. The two numbers after the second underline indicate how may rows and columns are in the plot.

## Value

The result is a plot

## Note

more notes

## Author(s)

Mikis Stasinopoulos, Robert Rigby, Gillian Heller, Fernada De Bastiani

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
count_1_31()
```

---

DBI *The Double binomial distribution*

---

## Description

The function DBI() defines the double binomial distribution, a two parameters distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDBI, pDBI, qDBI and rDBI define the density, distribution function, quantile function and random generation for the double binomial, DBI(), distribution. The function GetBI_C calculates numericaly the constant of proportionality needed for the pdf to sum up to 1.

## Usage

```
DBI(mu.link = "logit", sigma.link = "log")
dDBI(x, mu = 0.5, sigma = 1, bd = 2, log = FALSE)
pDBI(q, mu = 0.5, sigma = 1, bd = 2, lower.tail = TRUE,
       log.p = FALSE)
qDBI(p, mu = 0.5, sigma = 1, bd = 2, lower.tail = TRUE,
       log.p = FALSE)
rDBI(n, mu = 0.5, sigma = 1, bd = 2)
GetBI_C(mu, sigma, bd)
```

## Arguments

| | |
|---|---|
| mu.link | the link function for mu with default log |
| sigma.link | the link function for sigma with default log |
| x, q | vector of (non-negative integer) quantiles |
| bd | vector of binomial denominator |
| p | vector of probabilities |
| mu | the mu parameter |
| sigma | the sigma parameter |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| n | how many random values to generate |

## Details

The definition for the Double Poisson distribution first introduced by Efron (1986) is:

$$p_Y(y|n,\mu,\sigma) = \frac{1}{C(n,\mu,\sigma)} \frac{\Gamma(n+1)}{\Gamma(y+1)\Gamma(n-y+1)} \frac{y^y\,(n-y)^{n-y}}{n^n} \frac{n^{n/\sigma}\mu^{y/\sigma}\,(1-\mu)^{(n-y)/\sigma}}{y^{y/\sigma}\,(n-y)^{(n-y)/\sigma}}$$

for $y = 0, 1, 2, \ldots, \infty$, $\mu > 0$ and $\sigma > 0$ where $C$ is the constant of proportinality which is calculated numerically using the function GetBI_C(), see pp. 524-525 of Rigby *et al.* (2019).

## Value

The function DBI returns a gamlss.family object which can be used to fit a double binomial distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby, Marco Enea and Fernanda de Bastiani

## References

Efron, B., 1986. Double exponential families and their use in generalized linear Regression. Journal of the American Statistical Association 81 (395), 709-721.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

BI,BB

## Examples

```
DBI()
x <- 0:20
# underdispersed DBI
plot(x, dDBI(x, mu=.5, sigma=.2, bd=20), type="h", col="green", lwd=2)
# binomial
lines(x+0.1, dDBI(x, mu=.5, sigma=1, bd=20), type="h", col="black", lwd=2)
# overdispersed DBI
lines(x+.2, dDBI(x, mu=.5, sigma=2, bd=20), type="h", col="red",lwd=2)
```

---

DBURR12 *The Discrete Burr type XII distribution for fitting a GAMLSS model*

---

## Description

The DBURR12() function defines the discrete Burr type XII distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDBURR12(), pDBURR12(), qDBURR12() and rDBURR12() define the density, distribution function, quantile function and random generation for the discrete Burr type XII DBURR12(), distribution.

## Usage

```
DBURR12(mu.link = "log", sigma.link = "log", nu.link = "log")
dDBURR12(x, mu = 5, sigma = 2, nu = 2, log = FALSE)
pDBURR12(q, mu = 5, sigma = 2, nu = 2, lower.tail = TRUE,
        log.p = FALSE)
```

```
qDBURR12(p, mu = 5, sigma = 2, nu = 2, lower.tail = TRUE,
        log.p = FALSE)
rDBURR12(n, mu = 5, sigma = 2, nu = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter |
| x | vector of (non-negative integer) quantiles |
| p | vector of probabilities |
| q | vector of quantiles |
| mu | vector of positive mu |
| sigma | vector of positive dispersion parameter sigma |
| nu | vector of nu |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| n | number of random values to return |

## Details

The probability function of the discrete Burr XII distribution is given by

$$f(y|\mu,\sigma,\nu) = (1+(y/\mu)^{\sigma})^{\nu} - (1+((y+1)/\mu)^{\sigma})^{\nu}$$

for $y = 0, 1, 2, ..., \infty$, $\mu > 0$ , $\sigma > 0$ and $\mu > 0$ see pp 504-505 of Rigby *et al.* (2019).

Note that the above parametrization is different from Para and Jan (2016).

## Value

The function DBURR12() Returns a gamlss.family object which can be used to fit a discrete Burr XII distribution in the gamlss() function.

## Note

The parameters of the distributioins are highly correlated so the argument of gamlss method=mixed(10,100) may have to be used.

The distribution can be under/over dispersed and also with long tails.

## Author(s)

Rigby, R. A., Stasinopoulos D. M., Fernanda De Bastiani.

## References

Para, B. A. and Jan, T. R. (2016). On discrete three parameter Burr type XII and discrete Lomax distributions and their applications to model count data from medical science. *Biometrics and Biostatistics International Journal*, **54**, part 3, pp 507-554.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **4**, pp 1-15.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, DPO

## Examples

```
DBURR12()#
#plot the pdf using plot
plot(function(y) dDBURR12(y, mu=10, sigma=1, nu=1), from=0, to=100, n=100+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=100),pDBURR12(seq(from=0,to=100), mu=10, sigma=1, nu=1), type="h")  # cdf
# generate random sample
tN <- table(Ni <- rDBURR12(100, mu=5, sigma=1, nu=1))
r <- barplot(tN, col='lightblue')
```

---

DEL *The Delaporte distribution for fitting a GAMLSS model*

---

## Description

The DEL() function defines the Delaporte distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDEL, pDEL, qDEL and rDEL define the density, distribution function, quantile function and random generation for the Delaporte DEL(), distribution.

## Usage

```
DEL(mu.link = "log", sigma.link = "log", nu.link = "logit")
dDEL(x, mu=1, sigma=1, nu=0.5, log=FALSE)
pDEL(q, mu=1, sigma=1, nu=0.5, lower.tail = TRUE,
        log.p = FALSE)
```

```
qDEL(p, mu=1, sigma=1, nu=0.5,  lower.tail = TRUE,
     log.p = FALSE,  max.value = 10000)
rDEL(n, mu=1, sigma=1, nu=0.5, max.value = 10000)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "logit" link as the default for the nu parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive mu |
| sigma | vector of positive dispersion parameter |
| nu | vector of nu |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| max.value | a constant, set to the default value of 10000 for how far the algorithm should look for q |

## Details

The probability function of the Delaporte distribution is given by

$$f(y|\mu,\sigma,\nu) = \frac{e^{-\mu\nu}}{\Gamma(1/\sigma)}\left[1+\mu\sigma(1-\nu)\right]^{-1/\sigma} S$$

where

$$S = \sum_{j=0}^{y} \binom{y}{j} \frac{\mu^y \nu^{y-j}}{y!} \left[\mu + \frac{1}{\sigma(1-\nu)}\right]^{-j} \Gamma\left(\frac{1}{\sigma}+j\right)$$

for $y = 0, 1, 2, ..., \infty$ where $\mu > 0$ , $\sigma > 0$ and $0 < \nu < 1$. This distribution is a parametrization of the distribution given by Wimmer and Altmann (1999) p 515-516 where $\alpha = \mu\nu$, $k = 1/\sigma$ and $\rho = [1+\mu\sigma(1-\nu)]^{-1}$. For more details see pp 506-507 of Rigby *et al.* (2019).

## Value

Returns a gamlss.family object which can be used to fit a Delaporte distribution in the gamlss() function.

## Note

The mean of $Y$ is given by $E(Y) = \mu$ and the variance by $V(Y) = \mu + \mu^2\sigma\left(1-\nu\right)^2$.

## Author(s)

Rigby, R. A., Stasinopoulos D. M. and Marco Enea

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC,doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

Wimmer, G. and Altmann, G (1999). *Thesaurus of univariate discrete probability distributions* . Stamn Verlag, Essen, Germany

(see also https://www.gamlss.com/).

## See Also

gamlss.family, SI , SICHEL

## Examples

```
 DEL()# gives information about the default links for the  Delaporte distribution
#plot the pdf using plot
plot(function(y) dDEL(y, mu=10, sigma=1, nu=.5), from=0, to=100, n=100+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=100),pDEL(seq(from=0,to=100), mu=10, sigma=1, nu=0.5), type="h")  # cdf
# generate random sample
tN <- table(Ni <- rDEL(100, mu=10, sigma=1, nu=0.5))
r <- barplot(tN, col='lightblue')
# fit a model to the data
# libary(gamlss)
# gamlss(Ni~1,family=DEL, control=gamlss.control(n.cyc=50))
```

---

DPO *The Double Poisson distribution*

---

## Description

The function DPO() defines the double Poisson distribution, a two parameters distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDPO, pDPO, qDPO and rPO define the density, distribution function, quantile function and random generation for the double Poisson, DPO(), distribution. The function get_C() calculates numericaly the constant of proportionality needed for the pdf to sum up to 1.

## Usage

```
DPO(mu.link = "log", sigma.link = "log")
dDPO(x, mu = 1, sigma = 1, log = FALSE)
pDPO(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qDPO(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE,
     max.value = 10000)
rDPO(n, mu = 1, sigma = 1, max.value = 10000)
get_C(x, mu, sigma)
```

## Arguments

| | |
|---|---|
| mu.link | the link function for mu with default log |
| sigma.link | the link function for sigma with default log |
| x, q | vector of (non-negative integer) quantiles |
| p | vector of probabilities |
| mu | the mu parameter |
| sigma | the sigma parameter |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| max.value | a constant, set to the default value of 10000 for how far the algorithm should look for q |
| n | how many random values to generate |

## Details

The definition for the Double Poisson distribution first introduced by Efron (1986) [ see also Rigby et al (2019) page 480] is:

$$f(y|\mu,\sigma) = \left(\frac{1}{\sigma}\right)^{1/2} e^{-\mu/\sigma} \left(\frac{\mu}{y}\right)^{y/\sigma} \left(\frac{e^{y/\sigma - y}y^{y}}{y!}\right) C(\mu,\sigma)$$

for $y = 0, 1, 2, \ldots, \infty$, $\mu > 0$ and $\sigma > 0$ where $C$ is the constant of proportinality which is calculated numerically using the function get_C.

## Value

The function DPO returns a gamlss.family object which can be used to fit a double Poisson distribution in the gamlss() function.

## Note

The distributons calculates the constant of proportionality numerically therefore it can be slow for large data

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Marco Enea

## References

Efron, B., 1986. Double exponential families and their use in generalized linear Regression. Journal of the American Statistical Association 81 (395), 709-721.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

PO

## Examples

```
DPO()
# overdisperse DPO
x <- 0:20
plot(x, dDPO(x, mu=5, sigma=3), type="h", col="red")
# underdisperse DPO
plot(x, dDPO(x, mu=5, sigma=.3), type="h", col="red")
# generate random sample
 Y <- rDPO(100,5,.5)
plot(table(Y))
points(0:20, 100*dDPO(0:20, mu=5, sigma=.5)+0.2,  col="red")
# fit a model to the data
# library(gamlss)
# gamlss(Y~1,family=DPO)
```

---

# EGB2 *The exponential generalized Beta type 2 distribution for fitting a GAMLSS*

---

## Description

This function defines the generalized t distribution, a four parameter distribution. The response variable is in the range from minus infinity to plus infinity. The functions dEGB2, pEGB2, qEGB2 and rEGB2 define the density, distribution function, quantile function and random generation for the generalized beta type 2 distribution.

## Usage

```
EGB2(mu.link = "identity", sigma.link = "log", nu.link = "log",
      tau.link = "log")
dEGB2(x, mu = 0, sigma = 1, nu = 1, tau = 0.5, log = FALSE)
pEGB2(q, mu = 0, sigma = 1, nu = 1, tau = 0.5, lower.tail = TRUE,
      log.p = FALSE)
qEGB2(p, mu = 0, sigma = 1, nu = 1, tau = 0.5, lower.tail = TRUE,
      log.p = FALSE)
rEGB2(n, mu = 0, sigma = 1, nu = 1, tau = 0.5)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter. |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the Generalized Beta type 2, (GB2), is defined as:

$$f(y|\mu,\sigma\,\nu,\tau) = e^{\nu z}\left[|\sigma|B(\nu,\tau)\,(1+e^{z})^{\nu+\tau}\right]^{-1}$$

for $-\infty < y < \infty$, where $z = (y-\mu)/\sigma$ and $-\infty < \mu < \infty$, $-\infty < \sigma < \infty$, $\nu > 0$ and $\tau > 0$, McDonald and Xu (1995), see also pp. 385-386 of Rigby et al. (2019).

## Value

EGB2() returns a gamlss.family object which can be used to fit the EGB2 distribution in the gamlss() function. dEGB2() gives the density, pEGB2() gives the distribution function, qEGB2() gives the quantile function, and rEGB2() generates random deviates.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSU, BCT

## Examples

```
EGB2()   #
y<- rEGB2(200, mu=5, sigma=2, nu=1, tau=4)
library(MASS)
truehist(y)
fx<-dEGB2(seq(min(y), 20, length=200), mu=5 ,sigma=2, nu=1, tau=4)
lines(seq(min(y),20,length=200),fx)
# something funny here
# library(gamlss)
# histDist(y, family=EGB2, n.cyc=60)
integrate(function(x) x*dEGB2(x=x, mu=5, sigma=2, nu=1, tau=4), -Inf, Inf)
curve(dEGB2(x, mu=5 ,sigma=2, nu=1, tau=4), -10, 10, main = "The EGB2  density
            mu=5, sigma=2, nu=1, tau=4")
```

---

# exGAUS *The ex-Gaussian distribution*

---

## Description

The ex-Gaussian distribution is often used by psychologists to model response time (RT). It is defined by adding two random variables, one from a normal distribution and the other from an exponential. The parameters mu and sigma are the mean and standard deviation from the normal distribution variable while the parameter nu is the mean of the exponential variable. The functions dexGAUS, pexGAUS, qexGAUS and rexGAUS define the density, distribution function, quantile function and random generation for the ex-Gaussian distribution.

## Usage

```
exGAUS(mu.link = "identity", sigma.link = "log", nu.link = "log")
dexGAUS(x, mu = 5, sigma = 1, nu = 1, log = FALSE)
pexGAUS(q, mu = 5, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qexGAUS(p, mu = 5, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
rexGAUS(n, mu = 5, sigma = 1, nu = 1, ...)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter. Other links are "inverse", "identity", "logshifted" (shifted from one) and "own" |
| x, q | vector of quantiles |
| mu | vector of mu parameter values |
| sigma | vector of scale parameter values |
| nu | vector of nu parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| ... | for extra arguments |

## Details

The probability density function of the ex-Gaussian distribution, (exGAUS), is defined as

$$f(y|\mu,\sigma,\nu) = \frac{1}{\nu} e^{\frac{\mu-y}{\nu}+\frac{\sigma^2}{2\nu^2}} \Phi(\frac{y-\mu}{\sigma} - \frac{\sigma}{\nu})$$

where $\Phi$ is the cdf of the standard normal distribution, for $-\infty < y < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$ and $\nu > 0$ see pp. 372-373 of Rigby et al. (2019).

## Value

exGAUS() returns a gamlss.family object which can be used to fit ex-Gaussian distribution in the gamlss() function. dexGAUS() gives the density, pexGAUS() gives the distribution function, qexGAUS() gives the quantile function, and rexGAUS() generates random deviates.

## Note

The mean of the ex-Gaussian is $\mu + \nu$ and the variance is $\sigma^2 + \nu^2$.

## Author(s)

Mikis Stasinopoulos and Bob Rigby

## References

Cousineau, D. Brown, S. and Heathecote A. (2004) Fitting distributions using maximum likelihood: Methods and packages, *Behavior Research Methods, Instruments and Computers*, **46**, 742-756.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCCG, GA, IG LNO

## Examples

```
exGAUS()   #
y<- rexGAUS(100, mu=300, nu=100, sigma=35)
hist(y)
# library(gamlss)
# m1<-gamlss(y~1, family=exGAUS)
# plot(m1)
curve(dexGAUS(x, mu=300 ,sigma=35,nu=100), 100, 600,
 main = "The ex-GAUS  density mu=300 ,sigma=35,nu=100")
plot(function(x) pexGAUS(x, mu=300,sigma=35,nu=100), 100, 600,
 main = "The ex-GAUS  cdf mu=300, sigma=35, nu=100")
```

---

# EXP *Exponential distribution for fitting a GAMLSS*

---

## Description

The function EXP defines the exponential distribution, a one parameter distribution for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The mu parameter represents the mean of the distribution. The functions dEXP, pEXP, qEXP and rEXP define the density, distribution function, quantile function and random generation for the specific parameterization of the exponential distribution defined by function EXP.

## Usage

```
EXP(mu.link ="log")
dEXP(x, mu = 1, log = FALSE)
pEXP(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qEXP(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rEXP(n, mu = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The specific parameterization of the exponential distribution used in EXP is

$$f(y|\mu) = \frac{1}{\mu}\exp\left\{-\frac{y}{\mu}\right\}$$

for y>0, $\mu > 0$ see pp. 422-23 of Rigby et al. (2019).

## Value

EXP() returns a gamlss.family object which can be used to fit an exponential distribution in the gamlss() function. dEXP() gives the density, pEXP() gives the distribution function, qEXP() gives the quantile function, and rEXP() generates random deviates.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Nicoleta Motpan

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

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Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
y<-rEXP(1000,mu=1) # generates 1000 random observations
hist(y)
# library(gamlss)
# histDist(y, family=EXP)
```

---

`flexDist` *Non-parametric pdf from limited information data*

---

## Description

This is an attempt to create a distribution function if the only existing information is the quantiles or expectiles of the distribution.

## Usage

```
flexDist(quantiles = list(values=c(-1.96,0,1.96), prob=c(0.05, .50, 0.95)),
         expectiles = list(), lambda = 10,
         kappa = 10, delta = 1e-07, order = 3, n.iter = 200,
         plot = TRUE, no.inter = 100, lower = NULL,
         upper = NULL, perc.quant = 0.3, ...)
```

## Arguments

| | |
|---|---|
| `quantiles` | a list with components `values` and `prob` |
| `expectiles` | a list with components `values` and `prob` |
| `lambda` | smoothing parameter for the log-pdf |
| `kappa` | smoothing parameter for log concavity |
| `delta` | smoothing parameter for ridge penalty |
| `order` | the order of the penalty for log-pdf |
| `n.iter` | maximum number of iterations |
| `plot` | whether to plot the result |
| `no.inter` | How many discrete probabilities to evaluate |
| `lower` | the lower value of the x |
| `upper` | the upper value of the x |
| `perc.quant` | how far from the quantile should go out to define the limit of x if not set by `lower` or `upper` |
| `...` | additional arguments |

## Value

Returns a list with components

| | |
|---|---|
| pdf | the hights of the fitted pdf, the sum of it multiplied by the Dx should add up to 1 i.e. sum(object$pdf*diff(object$x)[1]) |
| cdf | the fitted cdf |
| x | the values of x where the discretise distribution is defined |
| pFun | the cdf of the fitted non-parametric distribution |
| qFun | the inverse cdf function of the fitted non-parametric distribution |
| rFun | a function to generate a random sample from the fitted non-parametric distribution |

## Author(s)

Mikis Stasinopoulos, Paul Eilers, Bob Rigby and Vlasios Voudouris

## References

Eilers, P. H. C., Voudouris, V., Rigby R. A., Stasinopoulos D. M. (2012) Estimation of nonparametric density from sparse summary information, under review.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), Appl. Statist., 54, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. Journal of Statistical Software, Vol. 23, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

histSmo

## Examples

```
# Normal
r1<-flexDist(quantiles=list(values=qNO(c(0.05, 0.25, 0.5,0.75, 0.95), mu=0,
             sigma=1), prob=c( 0.05, 0.25, 0.5,0.75,0.95 )),
             no.inter=200, lambda=10,  kappa=10, perc.quant=0.3)
# GAMMA
r1<-flexDist(quantiles=list(values=qGA(c(0.05,0.25, 0.5,0.75,0.95), mu=1,
       sigma=.8), prob=c(0.05,0.25, 0.5,0.75,0.95)),
       expectiles=list(values=1, prob=0.5),  lambda=10,
       kappa=10, lower=0, upper=5)#
```

GA *Gamma distribution for fitting a GAMLSS*

## Description

The function GA defines the gamma distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The parameterization used has the mean of the distribution equal to $\mu$ and the variance equal to $\sigma^2\mu^2$. The functions dGA, pGA, qGA and rGA define the density, distribution function, quantile function and random generation for the specific parameterization of the gamma distribution defined by function GA.

## Usage

```
GA(mu.link = "log", sigma.link ="log")
dGA(x, mu = 1, sigma = 1, log = FALSE)
pGA(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qGA(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rGA(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse", "identity" ans "own" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse", "identity" and "own" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The specific parameterization of the gamma distribution used in GA is

$$f(y|\mu,\sigma) = \frac{y^{(1/\sigma^2-1)}\exp[-y/(\sigma^2\mu)]}{(\sigma^2\mu)^{(1/\sigma^2)}\Gamma(1/\sigma^2)}$$

for $y > 0$, $\mu > 0$ and $\sigma > 0$, see pp. 423-424 of Rigby et al. (2019).

## Value

GA() returns a gamlss.family object which can be used to fit a gamma distribution in the gamlss() function. dGA() gives the density, pGA() gives the distribution function, qGA() gives the quantile function, and rGA() generates random deviates. The latest functions are based on the equivalent R functions for gamma distribution.

## Note

$\mu$ is the mean of the distribution in GA. In the function GA, $\sigma$ is the square root of the usual dispersion parameter for a GLM gamma model. Hence $\sigma\mu$ is the standard deviation of the distribution defined in GA.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
GA()# gives information about the default links for the gamma distribution
# dat<-rgamma(100, shape=1, scale=10) # generates 100 random observations
# fit a gamlss model
# gamlss(dat~1,family=GA)
# fits a constant for each parameter mu and sigma of the gamma distribution
newdata<-rGA(1000,mu=1,sigma=1) # generates 1000 random observations
hist(newdata)
rm(dat,newdata)
```

GAF *The Gamma distribution family*

## Description

The function GAF() defines a gamma distribution family, which has three parameters. This is not the generalised gamma distribution which is called GG. The third parameter here is to model the mean and variance relationship. The distribution can be fitted using the function gamlss(). The mean of GAF is equal to mu. The variance is equal to sigma^2*mu^nu so the standard deviation is sigma*mu^(nu/2). The function is design for cases where the variance is proportional to a power of the mean. This is an instance of the Taylor's power low, see Enki et al. (2017). The functions dGAF, pGAF, qGAF and rGAF define the density, distribution function, quantile function and random generation for the GAF parametrization of the gamma family.

## Usage

```
GAF(mu.link = "log", sigma.link = "log", nu.link = "identity")
dGAF(x, mu = 1, sigma = 1, nu = 2, log = FALSE)
pGAF(q, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE,
    log.p = FALSE)
qGAF(p, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE,
    log.p = FALSE)
rGAF(n, mu = 1, sigma = 1, nu = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link with "identity" link as the default for the nu parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of power parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parametrization of the gamma family given in the function GAF() is:

$$f(y|\mu,\sigma_1)=\frac{y^{(1/\sigma_1^2-1)}\exp[-y/(\sigma_1^2\mu)]}{(\sigma_1^2\mu)^{(1/\sigma^2)}\Gamma(1/\sigma^2)}$$

for $y>0$, $\mu>0$ where $\sigma_1=\sigma\mu^{(\nu/2)-1}$ $\sigma>0$ and $-\infty<\nu<\infty$ see pp. 442-443 of Rigby et al. (2019).

## Value

GAF() returns a gamlss.family object which can be used to fit the gamma family in the gamlss() function.

## Note

For the function GAF(), $\mu$ is the mean and $\sigma\mu^{\nu/2}$ is the standard deviation of the gamma family. The GAF is design for fitting regression type models where the variance is proportional to a power of the mean.

Note that because the high correlation between the sigma and the nu parameter the mixed() method should be used in the fitting.

## Author(s)

Mikis Stasinopoulos, Robert Rigby and Fernanda De Bastiani

## References

Enki, D G, Noufaily, A., Farrington, P., Garthwaite, P., Andrews, N. and Charlett, A. (2017) Taylor's power law and the statistical modelling of infectious disease surveillance data, Journal of the Royal Statistical Society: Series A (Statistics in Society), volume=180, number=1, pages=45-72.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07..

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, GA, GG

## Examples

```
GAF()
## Not run:
m1<-gamlss(y~poly(x,2),data=abdom,family=GAF, method=mixed(1,100),
           c.crit=0.00001)
# using RS()
m2<-gamlss(y~poly(x,2),data=abdom,family=GAF,  n.cyc=5000, c.crit=0.00001)
# the estimates of nu slightly different
fitted(m1, "nu")[1]
fitted(m2, "nu")[1]
# global deviance almost identical
AIC(m1, m2)

## End(Not run)
```

---

GAMLSS *Create a GAMLSS Distribution*

---

## Description

A single class and corresponding methods encompassing all distributions from the **gamlss.dist** package using the workflow from the **distributions3** package.

## Usage

```
GAMLSS(family, mu, sigma, tau, nu)
```

## Arguments

| | |
|---|---|
| family | character. Name of a GAMLSS family provided by **gamlss.dist**, e.g., NO or BI for the normal or binomial distribution, respectively. |
| mu | numeric. GAMLSS mu parameter. Can be a scalar or a vector or missing if not part of the family. |
| sigma | numeric. GAMLSS sigma parameter. Can be a scalar or a vector or missing if not part of the family. |
| tau | numeric. GAMLSS tau parameter. Can be a scalar or a vector or missing if not part of the family. |
| nu | numeric. GAMLSS nu parameter. Can be a scalar or a vector or missing if not part of the family. |

## Details

The S3 class GAMLSS provides a unified workflow based on the **distributions3** package (see Zeileis et al. 2022) for all distributions from the **gamlss.dist** package. The idea is that from fitted gamlss model objects predicted probability distributions can be obtained for which moments (mean, variance, etc.), probabilities, quantiles, etc. can be obtained with corresponding generic functions.

The constructor function GAMLSS sets up a distribution object, representing a distribution from the GAMLSS (generalized additive model of location, scale, and shape) framework by the corresponding parameters plus a family attribute, e.g., NO for the normal distribution or BI for the binomial distribution. There can be up to four parameters, called mu (often some sort of location parameter), sigma (often some sort of scale parameter), tau and nu (other parameters, e.g., capturing shape, etc.).

All parameters can also be vectors, so that it is possible to define a vector of GAMLSS distributions from the same family with potentially different parameters. All parameters need to have the same length or must be scalars (i.e., of length 1) which are then recycled to the length of the other parameters.

Note that not all distributions use all four parameters, i.e., some use just a subset. In that case, the corresponding arguments in GAMLSS should be unspecified, NULL, or NA.

For the GAMLSS distribution objects there is a wide range of standard methods available to the generics provided in the **distributions3** package: pdf and log_pdf for the (log-)density (PDF), cdf for the probability from the cumulative distribution function (CDF), quantile for quantiles, random for simulating random variables, and support for the support interval (minimum and maximum). Internally, these methods rely on the usual d/p/q/r functions provided in **gamlss.dist**, see the manual pages of the individual families. The methods is_discrete and is_continuous can be used to query whether the distributions are discrete on the entire support or continuous on the entire support, respectively.

See the examples below for an illustration of the workflow for the class and methods.

## Value

A GAMLSS object, inheriting from distribution.

## References

Zeileis A, Lang MN, Hayes A (2022). "distributions3: From Basic Probability to Probabilistic Regression." Presented at *useR! 2022 - The R User Conference*. Slides, video, vignette, code at https://www.zeileis.org/news/user2022/.

## See Also

gamlss.family

## Examples

```
## package and random seed
library("distributions3")
set.seed(6020)

## three Weibull distributions
X <- GAMLSS("WEI", mu = c(1, 1, 2), sigma = c(1, 2, 2))
X

## moments
mean(X)
variance(X)
```

```
## support interval (minimum and maximum)
support(X)
is_discrete(X)
is_continuous(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ], main = "WEI(1,1)")
hist(x[2, ], main = "WEI(1,2)")
hist(x[3, ], main = "WEI(2,2)")

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(2, 2, 1)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)
```

---

`gamlss.family` *Family Objects for fitting a GAMLSS model*

---

## Description

GAMLSS families are the current available distributions that can be fitted using the gamlss() function.

## Usage

```
gamlss.family(object,...)
as.gamlss.family(object)
as.family(object)
## S3 method for class 'gamlss.family'
print(x,...)
```

## Arguments

| | |
|---|---|
| object | a gamlss family object e.g. BCT |
| x | a gamlss family object e.g. BCT |
| ... | further arguments passed to or from other methods. |

## Details

There are several distributions available for the response variable in the gamlss function. The following table display their names and their abbreviations in R. Note that the different distributions can be fitted using their R abbreviations (and optionally excluding the brackets) i.e. family=BI(), family=BI are equivalent.

| Distributions | R names | No of parameters |
|---|---|---|
| Beta | BE() | 2 |
| Beta Binomial | BB() | 2 |
| Beta negative binomial | BNB() | 3 |
| Beta one inflated | BEOI() | 3 |
| Beta zero inflated | BEZI() | 3 |
| Beta inflated | BEINF() | 4 |
| Binomial | BI() | 1 |
| Box-Cox Cole and Green | BCCG() | 3 |
| Box-Cox Power Exponential | BCPE() | 4 |
| Box-Cox-t | BCT() | 4 |
| Delaport | DEL() | 3 |
| Discrete Burr XII | DBURR12() | 3 |
| Double Poisson | DPO() | 2 |
| Double binomial | DBI() | 2 |
| Exponential | EXP() | 1 |
| Exponential Gaussian | exGAUS() | 3 |
| Exponential generalized Beta type 2 | EGB2() | 4 |
| Gamma | GA() | 2 |
| Generalized Beta type 1 | GB1() | 4 |
| Generalized Beta type 2 | GB2() | 4 |
| Generalized Gamma | GG() | 3 |
| Generalized Inverse Gaussian | GIG() | 3 |

| | | |
|---|---|---|
| Generalized t | GT() | 4 |
| Geometric | GEOM() | 1 |
| Geometric (original) | GEOMo() | 1 |
| Gumbel | GU() | 2 |
| Inverse Gamma | IGAMMA() | 2 |
| Inverse Gaussian | IG() | 2 |
| Johnson's SU | JSU() | 4 |
| Logarithmic | LG() | 1 |
| Logistic | LO() | 2 |
| Logit-Normal | LOGITNO() | 2 |
| log-Normal | LOGNO() | 2 |
| log-Normal (Box-Cox) | LNO() | 3 (1 fixed) |
| Negative Binomial type I | NBI() | 2 |
| Negative Binomial type II | NBII() | 2 |
| Negative Binomial family | NBF() | 3 |
| Normal Exponential *t* | NET() | 4 (2 fixed) |
| Normal | NO() | 2 |
| Normal Family | NOF() | 3 (1 fixed) |
| Normal Linear Quadratic | LQNO() | 2 |
| Pareto type 2 | PARETO2() | 2 |
| Pareto type 2 original | PARETO2o() | 2 |
| Power Exponential | PE() | 3 |
| Power Exponential type 2 | PE2() | 3 |
| Poison | PO() | 1 |
| Poisson inverse Gaussian | PIG() | 2 |
| Reverse generalized extreme | RGE() | 3 |
| Reverse Gumbel | RG() | 2 |
| Skew Power Exponential type 1 | SEP1() | 4 |
| Skew Power Exponential type 2 | SEP2() | 4 |
| Skew Power Exponential type 3 | SEP3() | 4 |
| Skew Power Exponential type 4 | SEP4() | 4 |
| Shash | SHASH() | 4 |
| Shash original | SHASHo() | 4 |
| Shash original 2 | SHASH() | 4 |
| Sichel (original) | SI() | 3 |
| Sichel (mu as the maen) | SICHEL() | 3 |
| Simplex | SIMPLEX() | 2 |
| Skew t type 1 | ST1() | 3 |
| Skew t type 2 | ST2() | 3 |
| Skew t type 3 | ST3() | 3 |
| Skew t type 4 | ST4() | 3 |
| Skew t type 5 | ST5() | 3 |
| t-distribution | TF() | 3 |
| Waring | WARING() | 1 |
| Weibull | WEI() | 2 |
| Weibull(PH parameterization) | WEI2() | 2 |
| Weibull (mu as mean) | WEI3() | 2 |
| Yule | YULE() | 1 |

| | | |
|---|---|---|
| Zero adjusted binomial | ZABI() | 2 |
| Zero adjusted beta neg. bin. | ZABNB() | 4 |
| Zero adjusted IG | ZAIG() | 2 |
| Zero adjusted logarithmic | ZALG() | 2 |
| Zero adjusted neg. bin. | ZANBI() | 3 |
| Zero adjusted poisson | ZAP() | 2 |
| Zero adjusted Sichel | ZASICHEL() | 4 |
| Zero adjusted Zipf | ZAZIPF() | 2 |
| Zero inflated binomial | ZIBI() | 2 |
| Zero inflated beta neg. bin. | ZIBNB() | 4 |
| Zero inflated neg. bin. | ZINBI() | 3 |
| Zero inflated poisson | ZIP() | 2 |
| Zero inf. poiss.(mu as mean) | ZIP2() | 2 |
| Zero inflated PIG | ZIPIG() | 3 |
| Zero inflated Sichel | ZISICHEL() | 4 |
| Zipf | ZIPF() | 1 |

Note that some of the distributions are in the package `gamlss.dist`. The parameters of the distributions are in order, `mu` for location, `sigma` for scale (or dispersion), and `nu` and `tau` for shape. More specifically for the `BCCG` family `mu` is the median, `sigma` approximately the coefficient of variation, and `nu` the skewness parameter. The parameters for `BCPE` distribution have the same interpretation with the extra fourth parameter `tau` modelling the kurtosis of the distribution. The parameters for BCT have the same interpretation except that $\sigma[(\tau/(\tau-2))^{0.5}]$ is approximately the coefficient of variation.

All of the distribution in the above list are also provided with the corresponding `d`, `p`, `q` and `r` functions for density (pdf), distribution function (cdf), quantile function and random generation function respectively, (see individual distribution for details).

## Value

The above GAMLSS families return an object which is of type `gamlss.family`. This object is used to define the family in the `gamlss()` fit.

## Note

More distributions will be documented in later GAMLSS releases. Further user defined distributions can be incorporate relatively easy, see, for example, the help documentation accompanying the gamlss library.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

BE,BB,BEINF,BI,LNO,BCT, BCPE,BCCG, GA,GU,JSU,IG,LO, NBI,NBII,NO,PE,PO, RG,PIG,TF,WEI,WEI2, ZIP

## Examples

```
normal<-NO(mu.link="log", sigma.link="log")
normal
```

---

GB1 *The generalized Beta type 1 distribution for fitting a GAMLSS*

---

## Description

This function defines the generalized beta type 1 distribution, a four parameter distribution. The function GB1 creates a gamlss.family object which can be used to fit the distribution using the function gamlss(). Note the range of the response variable is from zero to one. The functions dGB1, GB1, qGB1 and rGB1 define the density, distribution function, quantile function and random generation for the generalized beta type 1 distribution.

## Usage

```
GB1(mu.link = "logit", sigma.link = "logit", nu.link = "log",
      tau.link = "log")
dGB1(x, mu = 0.5, sigma = 0.4, nu = 1, tau = 1, log = FALSE)
pGB1(q, mu = 0.5, sigma = 0.4, nu = 1, tau = 1, lower.tail = TRUE,
       log.p = FALSE)
qGB1(p, mu = 0.5, sigma = 0.4, nu = 1, tau = 1, lower.tail = TRUE,
       log.p = FALSE)
rGB1(n, mu = 0.5, sigma = 0.4, nu = 1, tau = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter. |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. |

| | |
|---|---|
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the Generalized Beta type 1, (GB1), is defined as

$$f(y|\mu,\sigma\,\nu,\tau) = \frac{\tau\nu^{\beta}y^{\tau\alpha-1}\left(1-y^{\tau}\right)^{\beta-1}}{B(\alpha,\beta)[\nu+(1-\nu)y^{\tau}]^{\alpha+\beta}}$$

where $0 < y < 1$, $0 < \mu < 1$, $0 < \sigma < 1$, $\nu > 0$, $\tau > 0$ and where $\alpha = \mu(1-\sigma^2)/\sigma^2$ and $\beta = (1-\mu)(1-\sigma^2)/\sigma^2$, and $\alpha > 0$, $\beta > 0$. Note the $\mu = \alpha/(\alpha+\beta)$, $\sigma = (\alpha+\beta+1)^{-1/2}$ see pp. 464-465 of Rigby et al. (2019).

## Value

GB1() returns a gamlss.family object which can be used to fit the GB1 distribution in the gamlss() function. dGB1() gives the density, pGB1() gives the distribution function, qGB1() gives the quantile function, and rGB1() generates random deviates.

## Warning

The qSHASH and rSHASH are slow since they are relying on golden section for finding the quantiles

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSU, BCT

## Examples

```
GB1()   #
y<- rGB1(200, mu=.1, sigma=.6, nu=1, tau=4)
hist(y)
# library(gamlss)
# histDist(y, family=GB1, n.cyc=60)
curve(dGB1(x, mu=.1 ,sigma=.6, nu=1, tau=4), 0.01, 0.99, main = "The GB1
           density mu=0.1, sigma=.6, nu=1, tau=4")
```

---

# GB2 *The generalized Beta type 2 and generalized Pareto distributions for fitting a GAMLSS*

---

## Description

This function defines the generalized beta type 2 distribution, a four parameter distribution. The function GB2 creates a gamlss.family object which can be used to fit the distribution using the function gamlss(). The response variable is in the range from zero to infinity. The functions dGB2, GB2, qGB2 and rGB2 define the density, distribution function, quantile function and random generation for the generalized beta type 2 distribution. The generalised Pareto GP distribution is defined by setting the parameters sigma and nu of the GB2 distribution to 1.

## Usage

```
GB2(mu.link = "log", sigma.link = "log", nu.link = "log",
     tau.link = "log")
dGB2(x, mu = 1, sigma = 1, nu = 1, tau = 0.5, log = FALSE)
pGB2(q, mu = 1, sigma = 1, nu = 1, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
qGB2(p, mu = 1, sigma = 1, nu = 1, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
rGB2(n, mu = 1, sigma = 1, nu = 1, tau = 0.5)

GP(mu.link = "log", sigma.link = "log")
dGP(x, mu = 1, sigma = 1, log = FALSE)
pGP(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qGP(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rGP(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter. |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the Generalized Beta type 2, (GB2), is defined as

$$f(y|\mu,\sigma,\nu,\tau) = |\sigma| y^{\sigma\nu-1}\{\mu^{\sigma\nu}\, B(\nu,\tau)[1+(y/\mu)^{\sigma}]^{\nu+\tau}\}^{-1}$$

where $y > 0$, $\mu > 0$, $\sigma > 0$, $\nu > 0$ and $\tau > 0$ see pp. 452-453 of Rigby et al. (2019).

Note that by setting $\sigma = 1$ we have the Pearson type VI, by setting $\nu = 1$ we have the Burr type XII and by setting $\tau = 1$ the Burr type III.

## Value

GB2() returns a gamlss.family object which can be used to fit the GB2 distribution in the gamlss() function. dGB2() gives the density, pGB2() gives the distribution function, qGB2() gives the quantile function, and rGB2() generates random deviates.

## Warning

The qSHASH and rSHASH are slow since they are relying on golden section for finding the quantiles

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSU, BCT

## Examples

```
GB2()   #
y<- rGB2(200, mu=5, sigma=2, nu=1, tau=1)
library(MASS)
truehist(y)
fx<-dGB2(seq(0.01, 20, length=200), mu=5 ,sigma=2, nu=1, tau=1)
lines(seq(0.01,20,length=200),fx)
integrate(function(x) x*dGB2(x=x, mu=5, sigma=2, nu=1, tau=1), 0, Inf)
mean(y)
curve(dGB2(x, mu=5 ,sigma=2, nu=1, tau=1), 0.01, 20,
            main = "The GB2  density mu=5, sigma=2, nu=1, tau=4")
```

---

# gen.Family *Functions to generate log and logit distributions from existing continuous gamlss.family distributions*

---

## Description

There are five functions here. Only the functions `Family` and `gen.Family` should be used (see details).

## Usage

```
Family.d(family = "NO", type = c("log", "logit"), ...)
Family.p(family = "NO", type = c("log", "logit"), ...)
Family.q(family = "NO", type = c("log", "logit"), ...)
Family.r(family = "NO", type = c("log", "logit"), ...)
Family(family = "NO", type = c("log", "logit"), local = TRUE, ...)
gen.Family(family = "NO", type = c("log", "logit"), ...)
```

## Arguments

| | |
|---|---|
| family | a continuous gamlss.family distribution |
| type | the type of transformation only "log" and "logit" are allowed |
| local | It is TRUE if is called within gamlss() otherwise is FALSE |
| ... | for passing extra arguments |

## Details

The function gen.Family creates the standard d,p,q,r functions for the distribution plus the fitting gamlss.family. For example gen.Family("NO", "logit") will generate the functions dlogitNO(), plogitNO() , qlogitNO() , rlogitNO() and dlogitNO(). The latest function can be used in family argument of gamlss() to fit a logic-Normal distribution i.e. family=logitNO. The same fitting can be achieved by using family=Family("NO", "logit"). Here the required dlogitNO(), plogitNO() and logitNO() functions are generated locally within the gamlss() environment.

## Value

The function gen.Family returns the d, p, q r functions plus the fitting function.

## Author(s)

Mikis Stasinopoulos and Bob Rigby

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## Examples

```
# generating a log t distribution
gen.Family("TF")
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dlogTF(x, mu=0), 0, 10)
curve(plogTF(x, mu=0), 0, 10)
curve(qlogTF(x, mu=0), 0, 1)
Y<- rlogTF(200)
hist(Y)
```

```
par(op)


# different mu
curve(dlogTF(x, mu=-1, sigma=1, nu=10), 0, 5, ylim=c(0,1))
curve(dlogTF(x, mu=0, sigma=1, nu=10), 0,  5, add=TRUE, col="red", lty=2)
curve(dlogTF(x, mu=1, sigma=1, nu=10), 0, 5, add=TRUE, col="blue", lty=3)

# different sigma
curve(dlogTF(x, mu=0, sigma=.5, nu=10), 0, 5, ylim=c(0,1))
curve(dlogTF(x, mu=0, sigma=1, nu=10), 0, 5, add=TRUE, col="red", lty=2)
curve(dlogTF(x, mu=0, sigma=2, nu=10), 0, 5, add=TRUE, col="blue", lty=3)

# different degrees of freedom nu
curve(dlogTF(x,  mu=0, sigma=1, nu=1), 0, 5, ylim=c(0,.8), n = 1001)
curve(dlogTF(x,  mu=0, sigma=1, nu=2), 0, 5, add=TRUE, col="red", lty=2)
curve(dlogTF(x,  mu=0, sigma=1, nu=5), 0, 5, add=TRUE, col="blue", lty=3)


# generating a logit t distribution
gen.Family("TF", "logit")
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dlogitTF(x, mu=0), 0, 1)
curve(plogitTF(x, mu=0), 0, 1)
curve(qlogitTF(x, mu=0), 0, 1)
abline(v=1)
Y<- rlogitTF(200)
hist(Y)
par(op)


# different mu
curve(dlogitTF(x, mu=-2, sigma=1, nu=10), 0, 1, ylim=c(0,5))
curve(dlogitTF(x, mu=0, sigma=1, nu=10), 0, 1, add=TRUE, col="red", lty=2)
curve(dlogitTF(x, mu=2, sigma=1, nu=10), 0, 1, add=TRUE, col="blue", lty=3)

# different sigma
curve(dlogitTF(x, mu=0, sigma=1, nu=10), 0, 1, ylim=c(0,2.5))
curve(dlogitTF(x, mu=0, sigma=2, nu=10), 0, 1, add=TRUE, col="red", lty=2)
curve(dlogitTF(x, mu=0, sigma=.7, nu=10), 0, 1, add=TRUE, col="blue", lty=3)

# different degrees of freedom nu
curve(dlogitTF(x,  mu=0, sigma=1, nu=1), 0, 1, ylim=c(0,1.6))
curve(dlogitTF(x,  mu=0, sigma=1, nu=2), 0, 1, add=TRUE, col="red", lty=2)
curve(dlogitTF(x,  mu=0, sigma=1, nu=5), 0, 1, add=TRUE, col="blue", lty=3)
```

---

GEOM *Geometric distribution for fitting a GAMLSS model*

---

## Description

The functions GEOMo() and GEOM() define two parametrizations of the geometric distribution. The geometric distribution is a one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The mean of GEOM() is equal to the parameter mu. The functions dGEOM, pGEOM, qGEOM and rGEOM define the density, distribution function, quantile function and random generation for the GEOM parameterization of the Geometric distribution.

## Usage

```
GEOM(mu.link = "log")
dGEOM(x, mu = 2, log = FALSE)
pGEOM(q, mu = 2, lower.tail = TRUE, log.p = FALSE)
qGEOM(p, mu = 2, lower.tail = TRUE, log.p = FALSE)
rGEOM(n, mu = 2)
GEOMo(mu.link = "logit")
dGEOMo(x, mu = 0.5, log = FALSE)
pGEOMo(q, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
qGEOMo(p, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
rGEOMo(n, mu = 0.5)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with log link as the default for the mu parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise P[X > x] |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the GAMLSS geometric distribution in the function GEOM is:

$$f(y|\mu) = \mu^y/(\mu+1)^{y+1}$$

for $y = 0, 1, 2, 3, ...$ and $\mu > 0$, see pp 472-473 of Rigby et al. (2019). Note that in this parametrization $E(y) = \mu$.

The parameterization of the original geometric distribution, GEOMo is

$$f(y|\mu) = (1-\mu)^y\mu$$

for $y = 0, 1, 2, 3, ...$ and $\mu > 0$, see pp 473-474 of Rigby et al. (2019). In this parametrization $E(y) = (1-\mu)\mu^{-1}$.

## Value

returns a `gamlss.family` object which can be used to fit a Geometric distribution in the `gamlss()` function.

## Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos.

## References

Johnson, N. L., Kemp, A. W., and Kotz, S. (2005). *Univariate discrete distributions.* Wiley.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, 54, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. 23, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
par(mfrow=c(2,2))
y<-seq(0,20,1)
plot(y, dGEOM(y), type="h")
q <- seq(0, 20, 1)
plot(q, pGEOM(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qGEOM(p), type="s")
dat <- rGEOM(100)
hist(dat)
#summary(gamlss(dat~1, family=GEOM))
par(mfrow=c(2,2))
y<-seq(0,20,1)
plot(y, dGEOMo(y), type="h")
q <- seq(0, 20, 1)
plot(q, pGEOMo(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qGEOMo(p), type="s")
dat <- rGEOMo(100)
hist(dat)
#summary(gamlss(dat~1, family="GE"))
```

GG *Generalized Gamma distribution for fitting a GAMLSS*

## Description

The function GG defines the generalized gamma distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The parameterization used has a rather compicated mean function which is depends on all the paramneters of the distribution, `mu`, `sigma` and `nu`, see page 444 of Rigby *et al.* (2019). The functions 'dGG', 'pGG', 'qGG' and 'rGG' define the density, distribution function, quantile function and random generation for the specific parameterization of the generalized gamma distribution defined by function GG.

## Usage

```
GG(mu.link = "log", sigma.link = "log",
                        nu.link = "identity")
dGG(x, mu=1, sigma=0.5, nu=1,
                       log = FALSE)
pGG(q, mu=1, sigma=0.5, nu=1,  lower.tail = TRUE,
                      log.p = FALSE)
qGG(p, mu=1, sigma=0.5, nu=1,  lower.tail = TRUE,
                      log.p = FALSE )
rGG(n, mu=1, sigma=0.5, nu=1)
```

## Arguments

| | |
|---|---|
| `mu.link` | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity" |
| `sigma.link` | Defines the sigma.link, with "log" link as the default for the sigma parameter, other links are "inverse" and "identity" |
| `nu.link` | Defines the nu.link, with "identity" link as the default for the sigma parameter, other links are $1/nu^2$ and "log" |
| `x`, `q` | vector of quantiles |
| `mu` | vector of location parameter values |
| `sigma` | vector of scale parameter values |
| `nu` | vector of shape parameter values |
| `log`, `log.p` | logical; if TRUE, probabilities p are given as log(p). |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| `p` | vector of probabilities |
| `n` | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The specific parameterization of the generalized gamma distribution used in GG is

$$f(y|\mu,\sigma,\nu) = \frac{\theta^\theta z^\theta |\nu| e^{-\theta z}}{(\Gamma(\theta) y)}$$

where $z = (y/\mu)^\nu$, $\theta = 1/(\sigma^2\nu^2)$ for y>0, $\mu > 0$, $\sigma > 0$ and $-\infty < \nu < +\infty$ see pp. 443-444 of Rigby et al. (2019). Note that for $\nu = 0$ the distribution is log normal.

## Value

GG() returns a gamlss.family object which can be used to fit a generalized gamma distribution in the gamlss() function. dGG() gives the density, pGG() gives the distribution function, qGG() gives the quantile function, and rGG() generates random deviates.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Nicoleta Motpan

## References

Lopatatzidis, A. and Green, P. J. (2000), Nonparametric quantile regression using the gamma distribution, unpublished.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, GA

## Examples

```
y<-rGG(100,mu=1,sigma=0.1, nu=-.5) # generates 100 random observations
hist(y)
# library(gamlss)
#histDist(y, family=GG)
#m1 <-gamlss(y~1,family=GG)
#prof.dev(m1, "nu", min=-2, max=2, step=0.2)
```

GIG *Generalized Inverse Gaussian distribution for fitting a GAMLSS*

## Description

The function GIG defines the generalized inverse gaussian distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions DIG, pGIG, GIG and rGIG define the density, distribution function, quantile function and random generation for the specific parameterization of the generalized inverse gaussian distribution defined by function GIG.

## Usage

```
GIG(mu.link = "log", sigma.link = "log",
                      nu.link = "identity")
dGIG(x, mu=1, sigma=1, nu=1,
                     log = FALSE)
pGIG(q, mu=1, sigma=1, nu=1,  lower.tail = TRUE,
                    log.p = FALSE)
qGIG(p, mu=1, sigma=1, nu=1,  lower.tail = TRUE,
                    log.p = FALSE)
rGIG(n, mu=1, sigma=1, nu=1, ...)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other links are "inverse" and "identity" |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter, other links are "inverse" and "log" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of shape parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| ... | for extra arguments |

## Details

The specific parameterization of the generalized inverse gaussian distribution used in GIG is

$$f(y|\mu,\sigma,\nu) = (\frac{b}{\mu})^{\nu}\left[\frac{y^{\nu-1}}{2K_{\nu}(\sigma^{-2})}\right]\exp\left[-\frac{1}{2\sigma^2}\left(\frac{by}{\mu}+\frac{\mu}{by}\right)\right]$$

where $b = \frac{K_{\nu+1}(\frac{1}{\sigma^2})}{K_{\nu}(\frac{1}{\sigma^2})}$, for y>0, $\mu > 0$, $\sigma > 0$ and $-\infty < \nu < +\infty$ see pp 445-446 of Rigby et al. (2019).

## Value

GIG() returns a gamlss.family object which can be used to fit a generalized inverse gaussian distribution in the gamlss() function. DIG() gives the density, pGIG() gives the distribution function, GIG() gives the quantile function, and rGIG() generates random deviates.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Nicoleta Motpan

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

Jorgensen B. (1982) Statistical properties of the generalized inverse Gaussian distribution, Series: Lecture notes in statistics; 9, New York : Springer-Verlag.

(see also https://www.gamlss.com/).

## See Also

gamlss.family, IG

## Examples

```
y<-rGIG(100,mu=1,sigma=1, nu=-0.5) # generates 1000 random observations
hist(y)
# library(gamlss)
# histDist(y, family=GIG)
```

GPO *The generalised Poisson distribution*

## Description

The GPO() function defines the generalised Poisson distribution, a two parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dGPO, pGPO, qGPO and rGPO define the density, distribution function, quantile function and random generation for the Delaporte GPO(), distribution.

## Usage

```
GPO(mu.link = "log", sigma.link = "log")

dGPO(x, mu = 1, sigma = 1, log = FALSE)

pGPO(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)

qGPO(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE, max.value = 10000)

rGPO(n, mu = 1, sigma = 1,  max.value = 10000)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive mu |
| sigma | vector of positive dispersion parameter sigma |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| max.value | a constant, set to the default value of 10000 for how far the algorithm should look for q |

## Details

The probability function of this parametrization of the Generalised Poisson distribution is given by

$$P(Y=y|\mu,\sigma)=\left(\frac{\mu}{1+\sigma\mu}\right)^{y}\frac{(1+\sigma y)^{y-1}}{y!}\exp\left[\frac{-\mu\,(1+\sigma y)}{1+\sigma\mu}\right]$$

for $y = 0, 1, 2, ..., \infty$ where $\mu > 0$ and $\sigma > 0$ see pp. 481-483 of Rigby *et al.* (2019). The advantage of this parametrisation is that $E(y) = \mu$ but the disadvantage is that we can NOT model underdispersion. For undedispersion see the `DBURR12` distribution.

## Value

Returns a `gamlss.family` object which can be used to fit a Generalised Poisson distribution in the `gamlss()` function.

## Author(s)

Rigby, R. A., Stasinopoulos D. M.

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

`gamlss.family`, `PO` , `DPO`

## Examples

```
GPO()# gives information about the default links for the
#plot the pdf using plot
plot(function(y) dGPO(y, mu=10, sigma=1 ), from=0, to=100, n=100+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=100),pGPO(seq(from=0,to=100), mu=10, sigma=1), type="h")   # cdf
# generate random sample
tN <- table(Ni <- rGPO(100, mu=5, sigma=1))
r <- barplot(tN, col='lightblue')
```

GT *The generalized t distribution for fitting a GAMLSS*

## Description

This function defines the generalized t distribution, a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dGT, pGT, qGT and rGT define the density, distribution function, quantile function and random generation for the generalized t distribution.

## Usage

```
GT(mu.link = "identity", sigma.link = "log", nu.link = "log",
   tau.link = "log")
dGT(x, mu = 0, sigma = 1, nu = 3, tau = 1.5, log = FALSE)
pGT(q, mu = 0, sigma = 1, nu = 3, tau = 1.5, lower.tail = TRUE,
   log.p = FALSE)
qGT(p, mu = 0, sigma = 1, nu = 3, tau = 1.5, lower.tail = TRUE,
   log.p = FALSE)
rGT(n, mu = 0, sigma = 1, nu = 3, tau = 1.5)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter. |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the generalized t distribution, (GT), , is defined as

$$f(y|\mu,\sigma\,\nu,\tau) = \tau\left\{2\sigma\nu^{1/\tau}B\left(\frac{1}{\tau},\nu\right)[1+|z|^{\tau}/\nu]^{\nu+1/\tau}\right\}^{-1}$$

where $-\infty < y < \infty$, $z = (y-\mu)/\sigma$ $\mu = (-\infty, +\infty)$, $\sigma > 0$, $\nu > 0$ and $\tau > 0$, see pp. 387-388 of Rigby et al. (2019).

## Value

`GT()` returns a `gamlss.family` object which can be used to fit the GT distribution in the `gamlss()` function. `dGT()` gives the density, `pGT()` gives the distribution function, `qGT()` gives the quantile function, and `rGT()` generates random deviates.

## Warning

The qGT and rGT are slow since they are relying on optimization for finding the quantiles

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSU, BCT

## Examples

```
GT()   #
y<- rGT(200, mu=5, sigma=1, nu=1, tau=4)
hist(y)
curve(dGT(x, mu=5 ,sigma=2,nu=1, tau=4), -2, 11,
      main = "The GT  density mu=5 ,sigma=1, nu=1, tau=4")
# library(gamlss)
# m1<-gamlss(y~1, family=GT)
```

GU *The Gumbel distribution for fitting a GAMLSS*

## Description

The function GU defines the Gumbel distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dGU, pGU, qGU and rGU define the density, distribution function, quantile function and random generation for the specific parameterization of the Gumbel distribution.

## Usage

```
GU(mu.link = "identity", sigma.link = "log")
dGU(x, mu = 0, sigma = 1, log = FALSE)
pGU(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qGU(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rGU(n, mu = 0, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. other available link is "inverse", "log" and "own") |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other links are the "inverse", "identity" and "own" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The specific parameterization of the Gumbel distribution used in GU is

$$f(y|\mu,\sigma) = \frac{1}{\sigma}\ \exp\left\{\left(\frac{y-\mu}{\sigma}\right) - \exp\left(\frac{y-\mu}{\sigma}\right)\right\}$$

for $y = (-\infty, \infty)$, $\mu = (-\infty, +\infty)$ and $\sigma > 0$, see pp. 366-367 of Rigby et al. (2019).

## Value

GU() returns a gamlss.family object which can be used to fit a Gumbel distribution in the gamlss() function. dGU() gives the density, pGU() gives the distribution function, qGU() gives the quantile function, and rGU() generates random deviates.

## Note

The mean of the distribution is $\mu - 0.57722\sigma$ and the variance is $\pi^2\sigma^2/6$.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, RG

## Examples

```
plot(function(x) dGU(x, mu=0,sigma=1), -6, 3,
 main = "{Gumbel  density mu=0,sigma=1}")
GU()# gives information about the default links for the Gumbel distribution
dat<-rGU(100, mu=10, sigma=2) # generates 100 random observations
hist(dat)
# library(gamlss)
# gamlss(dat~1,family=GU) # fits a constant for each parameter mu and sigma
```

---

# hazardFun *Hazard functions for gamlss.family distributions*

---

## Description

The function hazardFun() takes as an argument a gamlss.family object and creates the hazard function for it. The function gen.hazard() generates a hazard function called hNAME where NAME is a gamlss.family i.e. hGA().

## Usage

```
hazardFun(family = "NO", ...)
gen.hazard(family = "NO", ...)
```

## Arguments

| | |
|---|---|
| family | a gamlss.family object |
| ... | for passing extra arguments |

## Value

A hazard function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Vlasios Voudouris

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
gen.hazard("WEI2")
y<-seq(0,10,by=0.01)
plot(hWEI2(y, mu=1, sigma=1)~y, type="l", col="black", ylab="h(y)", ylim=c(0,2.5))
lines(hWEI2(y, mu=1, sigma=1.2)~y, col="red",lt=2,lw=2)
lines(hWEI2(y, mu=1, sigma=.5)~y, col="blue",lt=3,lw=2)
```

IG *Inverse Gaussian distribution for fitting a GAMLSS*

## Description

The function IG(), or equivalently Inverse.Gaussian(), defines the inverse Gaussian distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dIG, pIG, qIG and rIG define the density, distribution function, quantile function and random generation for the specific parameterization of the Inverse Gaussian distribution defined by function IG.

## Usage

```
IG(mu.link = "log", sigma.link = "log")
dIG(x, mu = 1, sigma = 1, log = FALSE)
pIG(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qIG(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rIG(n, mu = 1, sigma = 1, ...)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| ... | ... can be used to pass the uppr.limit argument to qIG |

## Details

Definition file for inverse Gaussian distribution.

$$f(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2 y^3}} \exp\left\{-\frac{1}{2\mu^2\sigma^2 y}(y-\mu)^2\right\}$$

for $y > 0$, $\mu > 0$ and $\sigma > 0$ see pp. 426-427 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a inverse Gaussian distribution in the gamlss() function.

## Note

$\mu$ is the mean and $\sigma^2\mu^3$ is the variance of the inverse Gaussian

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family , GA, GIG

## Examples

```
IG()# gives information about the default links for the normal distribution
# library(gamlss)
# data(rent)
# gamlss(R~cs(Fl),family=IG, data=rent) #
plot(function(x)dIG(x, mu=1,sigma=.5), 0.01, 6,
 main = "{Inverse Gaussian  density mu=1,sigma=0.5}")
plot(function(x)pIG(x, mu=1,sigma=.5), 0.01, 6,
 main = "{Inverse Gaussian  cdf mu=1,sigma=0.5}")
```

---

# IGAMMA *Inverse Gamma distribution for fitting a GAMLSS*

---

## Description

The function IGAMMA() defines the Inverse Gamma distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with parameters mu (the mode) and sigma. The functions dIGAMMA, pIGAMMA, qIGAMMA and rIGAMMA define the density, distribution function, quantile function and random generation for the IGAMMA parameterization of the Inverse Gamma distribution.

## Usage

```
IGAMMA(mu.link = "log", sigma.link="log")
dIGAMMA(x, mu = 1, sigma = .5, log = FALSE)
pIGAMMA(q, mu = 1, sigma = .5, lower.tail = TRUE, log.p = FALSE)
qIGAMMA(p, mu = 1, sigma = .5, lower.tail = TRUE, log.p = FALSE)
rIGAMMA(n, mu = 1, sigma = .5)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with log link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with log as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise P[X > x] |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the Inverse Gamma distribution in the function IGAMMA is

$$f(y|\mu,\sigma) = \frac{[\mu\,(\alpha+1)]^{\alpha}}{\Gamma(\alpha)}\, y^{-(\alpha+1)}\, \exp\left[-\frac{\mu\,(\alpha+1)}{y}\right]$$

where $\alpha = 1/(\sigma^2)$ for $y > 0$, $\mu > 0$ and $\sigma > 0$ see pp. 424-426 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit an Inverse Gamma distribution in the gamlss() function.

## Note

For the function IGAMMA(), *mu* is the mode of the Inverse Gamma distribution.

## Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos.

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, 54, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. 23, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, GA

## Examples

```
par(mfrow=c(2,2))
y<-seq(0.2,20,0.2)
plot(y, dIGAMMA(y), type="l")
q <- seq(0.2, 20, 0.2)
plot(q, pIGAMMA(q), type="l")
p<-seq(0.0001,0.999,0.05)
plot(p , qIGAMMA(p), type="l")
dat <- rIGAMMA(50)
hist(dat)
#summary(gamlss(dat~1, family="IGAMMA"))
```

---

# JSU *The Johnson's Su distribution for fitting a GAMLSS*

---

## Description

This function defines the , a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dJSU, pJSU, qJSU and rJSU define the density, distribution function, quantile function and random generation for the the Johnson's Su distribution.

## Usage

```
JSU(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log")
dJSU(x, mu = 0, sigma = 1, nu = 1, tau = 1, log = FALSE)
pJSU(q, mu = 0, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE)
qJSU(p, mu = 0, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE)
rJSU(n, mu = 0, sigma = 1, nu = 1, tau = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse" "log" ans "own" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" ans "own" |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "onverse", "log" and "own" |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "onverse", "identity" ans "own" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the Jonhson's SU distribution, (JSU), is defined as

$$f(y|\mu,\sigma\,\nu,\tau) = \frac{\tau}{c\sigma(s^2+1)^{\frac{1}{2}}\sqrt{2\pi}}\ \exp\left[-\frac{1}{2}z^2\right]$$

for $-\infty < y < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$, $-\infty < \nu < \infty$) and $\tau > 0$ and where

$$z = -\nu + \tau \sinh^{-1}(s)$$

,

$$s = \frac{y-\mu+c\sigma w^{\frac{1}{2}}\sinh(\nu/\tau)}{c\sigma}$$

,

$$c = \left\{\frac{1}{2}(w-1)\left[w\cosh(2\nu/\tau)+1\right]\right\}^{-\frac{1}{2}}$$

and

$$w = e^{1/\tau^2}$$

see pp. 393-394 of Rigby et al. (2019).

This is a reparameterization of the original Johnson Su distribution, Johnson (1954), so the parameters mu and sigma are the mean and the standard deviation of the distribution. The parameter nu

determines the skewness of the distribution with nu>0 indicating positive skewness and nu<0 negative. The parameter tau determines the kurtosis of the distribution. tau should be positive and most likely in the region from zero to 1. As tau goes to 0 (and for nu=0) the distribution approaches the the Normal density function. The distribution is appropriate for leptokurtic data that is data with kurtosis larger that the Normal distribution one.

## Value

JSU() returns a gamlss.family object which can be used to fit a Johnson's Su distribution in the gamlss() function. dJSU() gives the density, pJSU() gives the distribution function, qJSU() gives the quantile function, and rJSU() generates random deviates.

## Warning

The function JSU uses first derivatives square in the fitting procedure so standard errors should be interpreted with caution

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Johnson, N. L. (1954). Systems of frequency curves derived from the first law of Laplace., *Trabajos de Estadistica*, **5**, 283-291.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R. A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSUo, BCT

## Examples

```
JSU()
plot(function(x)dJSU(x, mu=0,sigma=1,nu=-1, tau=.5), -4, 4,
 main = "The JSU  density mu=0,sigma=1,nu=-1, tau=.5")
```

```
plot(function(x) pJSU(x, mu=0,sigma=1,nu=-1, tau=.5), -4, 4,
 main = "The JSU  cdf mu=0, sigma=1, nu=-1, tau=.5")
# library(gamlss)
# data(abdom)
# h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=JSU, data=abdom)
```

---

JSUo *The original Johnson's Su distribution for fitting a GAMLSS*

---

## Description

This function defines the , a four parameter distribution, for a `gamlss.family` object to be used for a GAMLSS fitting using the function `gamlss()`. The functions `dJSUo`, `pJSUo`, `qJSUo` and `rJSUo` define the density, distribution function, quantile function and random generation for the the Johnson's Su distribution.

## Usage

```
JSUo(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log")
dJSUo(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)
pJSUo(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)
qJSUo(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)
rJSUo(n, mu = 0, sigma = 1, nu = 0, tau = 1)
```

## Arguments

| | |
|---|---|
| `mu.link` | Defines the `mu.link`, with "identity" link as the default for the `mu` parameter. Other links are "inverse", "log" and "own" |
| `sigma.link` | Defines the `sigma.link`, with "log" link as the default for the `sigma` parameter. Other links are "inverse", "identity" and "own" |
| `nu.link` | Defines the `nu.link`, with "identity" link as the default for the `nu` parameter. Other links are "inverse", "log" ans "own" |
| `tau.link` | Defines the `tau.link`, with "log" link as the default for the `tau` parameter. Other links are "inverse", "identity" and "own" |
| `x`, `q` | vector of quantiles |
| `mu` | vector of location parameter values |
| `sigma` | vector of scale parameter values |
| `nu` | vector of skewness `nu` parameter values |
| `tau` | vector of kurtosis `tau` parameter values |
| `log`, `log.p` | logical; if TRUE, probabilities p are given as log(p). |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| `p` | vector of probabilities. |
| `n` | number of observations. If `length(n) > 1`, the length is taken to be the number required |

## Details

The probability density function of the orininal Jonhson's SU distribution, (JSUo), is defined as

$$f(y|\mu,\sigma\,\nu,\tau) = \frac{\tau}{\sigma(z^2+1)^{\frac{1}{2}}\sqrt{2\pi}}\ \exp\left[-\frac{1}{2}r^2\right]$$

for $-\infty < y < \infty$, $\mu = (-\infty, +\infty)$, $\sigma > 0$, $\nu = (-\infty, +\infty)$ and $\tau > 0$. where $z = \frac{(y-\mu)}{\sigma}$, $r = \nu + \tau \sinh^{-1}(z)$, see pp. 389-390 of Rigby et al. (2019).

## Value

JSUo() returns a gamlss.family object which can be used to fit a Johnson's Su distribution in the gamlss() function. dJSUo() gives the density, pJSUo() gives the distribution function, qJSUo() gives the quantile function, and rJSUo() generates random deviates.

## Warning

The function JSU uses first derivatives square in the fitting procedure so standard errors should be interpreted with caution. It is recomented to be used only with method=mixed(2,20)

## Author(s)

Mikis Stasinopoulos and Bob Rigby

## References

Johnson, N. L. (1954). Systems of frequency curves derived from the first law of Laplace., *Trabajos de Estadistica*, **5**, 283-291.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSU, BCT

## Examples

```
JSU()
plot(function(x)dJSUo(x, mu=0,sigma=1,nu=-1, tau=.5), -4, 15,
 main = "The JSUo  density mu=0,sigma=1,nu=-1, tau=.5")
plot(function(x) pJSUo(x, mu=0,sigma=1,nu=-1, tau=.5), -4, 15,
 main = "The JSUo  cdf mu=0, sigma=1, nu=-1, tau=.5")
# library(gamlss)
# data(abdom)
# h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=JSUo,
#          data=abdom, method=mixed(2,20))
# plot(h)
```

---

LG *Logarithmic and zero adjusted logarithmic distributions for fitting a GAMLSS model*

---

## Description

The function LG defines the logarithmic distribution, a one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dLG, pLG, qLG and rLG define the density, distribution function, quantile function and random generation for the logarithmic , LG(), distribution.

The function ZALG defines the zero adjusted logarithmic distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZALG, pZALG, qZALG and rZALG define the density, distribution function, quantile function and random generation for the inflated logarithmic , ZALG(), distribution.

## Usage

```
LG(mu.link = "logit")
dLG(x, mu = 0.5, log = FALSE)
pLG(q, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
qLG(p, mu = 0.5, lower.tail = TRUE, log.p = FALSE, max.value = 10000)
rLG(n, mu = 0.5)
ZALG(mu.link = "logit", sigma.link = "logit")
dZALG(x, mu = 0.5, sigma = 0.1, log = FALSE)
pZALG(q, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZALG(p, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZALG(n, mu = 0.5, sigma = 0.1)
```

## Arguments

| | |
|---|---|
| mu.link | defines the mu.link, with logit link as the default for the mu parameter |
| sigma.link | defines the sigma.link, with logit link as the default for the sigma parameter which in this case is the probability at zero. |
| x | vector of (non-negative integer) |

| | |
|---|---|
| mu | vector of positive means |
| sigma | vector of probabilities at zero |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| max.value | valued needed for the numerical calculation of the q-function |

## Details

The parameterization of the logarithmic distribution in the function LG is

$$P(Y = y|\mu) = \alpha\mu^y/y$$

where for $y = 1, 2, 3, ...$ with $0 < \mu < 1$ and

$$\alpha = -[\log(1 - \mu)]^{-1}.$$

with $E(y) = \alpha\mu(1 - \mu)^{-1}$ see pp 474-475 of Rigby *et al.* (2019).

For the zero adjusted logarithmic distribution ZALG which is defined for $y = 0, 1, 2, 3, ...$ see pp 492-494 of Rigby *et al.* (2019).

## Value

The function LG and ZALG return a gamlss.family object which can be used to fit a logarithmic and a zero inflated logarithmic distributions respectively in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Johnson, Norman Lloyd; Kemp, Adrienne W; Kotz, Samuel (2005). "Chapter 7: Logarithmic and Lagrangian distributions". Univariate discrete distributions (3 ed.). John Wiley & Sons. ISBN 9780471272465.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Rigby, R. A. and Stasinopoulos D. M. (2010) The gamlss.family distributions, (distributed with this package or see https://www.gamlss.com/)

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, PO, ZAP

## Examples

```
LG()
ZAP()
# creating data and plotting them
 dat <- rLG(1000, mu=.3)
   r <- barplot(table(dat), col='lightblue')
dat1 <- rZALG(1000, mu=.3, sigma=.1)
  r1 <- barplot(table(dat1), col='lightblue')
```

---

LNO *Log Normal distribution for fitting in GAMLSS*

---

## Description

The functions LOGNO and LOGNO2 define a gamlss.family distribution to fits the log-Normal distribution. The difference between them is that while LOGNO retains the original parametrization for mu, (identical to the normal distribution NO) and therefore $\mu = (-\infty, +\infty)$, the function LOGNO2 use mu as the median, so $\mu = (0, +\infty)$.

The function LNO is more general and can fit a Box-Cox transformation to data using the gamlss() function. In the LOGNO (and LOGNO2) there are two parameters involved mu sigma, while in the LNO there are three parameters mu sigma, and the transformation parameter nu. The transformation parameter nu in LNO is a 'fixed' parameter (not estimated) and it has its default value equal to zero allowing the fitting of the log-normal distribution as in LOGNO. See the example below on how to fix nu to be a particular value. In order to estimate (or model) the parameter nu, use the gamlss.family BCCG distribution which uses a reparameterized version of the the Box-Cox transformation. The functions dLOGNO, pLOGNO, qLOGNO and rLOGNO define the density, distribution function, quantile function and random generation for the specific parameterization of the log-normal distribution.

The functions dLOGNO2, pLOGNO2, qLOGNO2 and rLOGNO2 define the density, distribution function, quantile function and random generation when mu is the median of the log-normal distribution.

The functions dLNO, pLNO, qLNO and rLNO define the density, distribution function, quantile function and random generation for the specific parameterization of the log-normal distribution and more generally a Box-Cox transformation.

## Usage

```
LNO(mu.link = "identity", sigma.link = "log")
LOGNO(mu.link = "identity", sigma.link = "log")
LOGNO2(mu.link = "log", sigma.link = "log")
dLNO(x, mu = 1, sigma = 0.1, nu = 0, log = FALSE)
dLOGNO(x, mu = 0, sigma = 1, log = FALSE)
dLOGNO2(x, mu = 1, sigma = 1, log = FALSE)
```

```
pLNO(q, mu = 1, sigma = 0.1, nu = 0, lower.tail = TRUE, log.p = FALSE)
pLOGNO(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
pLOGNO2(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLNO(p, mu = 1, sigma = 0.1, nu = 0, lower.tail = TRUE, log.p = FALSE)
qLOGNO(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLOGNO2(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rLNO(n, mu = 1, sigma = 0.1, nu = 0)
rLOGNO(n, mu = 0, sigma = 1)
rLOGNO2(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" or "log" link depending on te parametrization |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" ans "own" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of shape parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function in LOGNO is defined as

$$f(y|\mu,\sigma) = \frac{1}{y\sqrt{2\pi}\sigma}\exp[-\frac{1}{2\sigma^2}(\log y - \mu)^2]$$

for $y > 0$, $-\infty < \mu < \infty$ and $\sigma > 0$ see pp. 428-429 of Rigby et al. (2019).

The probability density function in LOGNO2 is defined as

$$f(y|\mu,\sigma) = \frac{1}{y\sqrt{2\pi}\sigma}\exp[-\frac{1}{2\sigma^2}(\log y - \log \mu)^2]$$

for $y > 0$, $-\infty < \mu < \infty$ and $\sigma > 0$ see pp. 429-430 of Rigby et al. (2019).

The probability density function in LNO is defined as

$$f(y|\mu,\sigma,\nu) = \frac{y^{\nu-1}}{\sqrt{2\pi}\sigma}\exp[-\frac{1}{2\sigma^2}(z-\mu)^2]$$

where if $\nu \neq 0$ $z = (y^\nu - 1)/\nu$ else $z = \log(y)$ and $z \sim N(0,\sigma^2)$, for $y > 0$, $\mu > 0$, $\sigma > 0$ and $\nu = (-\infty, +\infty)$. This is not a proper distribution see for example p. 447 of Rigby et al. (2019).

## Value

LNO() returns a gamlss.family object which can be used to fit a log-normal distribution in the gamlss() function. dLNO() gives the density, pLNO() gives the distribution function, qLNO() gives the quantile function, and rLNO() generates random deviates.

## Warning

This is a two parameter fit for $\mu$ and $\sigma$ while $\nu$ is fixed. If you wish to model $\nu$ use the gamlss family BCCG.

## Note

$\mu$ is the mean of z (and also the median of y), the Box-Cox transformed variable and $\sigma$ is the standard deviation of z and approximate the coefficient of variation of y

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Box, G. E. P. and Cox, D. R. (1964) An analysis of transformations (with discussion), *J. R. Statist. Soc.* B., **26**, 211–252

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547.An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCCG

## Examples

```
LOGNO()#  gives information about the default links for the log normal distribution
LOGNO2()
LNO()# gives information about the default links for the Box Cox distribution

# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dLOGNO(x, mu=0), 0, 10)
```

```
curve(pLOGNO(x, mu=0), 0, 10)
curve(qLOGNO(x, mu=0), 0, 1)
Y<- rLOGNO(200)
hist(Y)
par(op)

# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dLOGNO2(x, mu=1), 0, 10)
curve(pLOGNO2(x, mu=1), 0, 10)
curve(qLOGNO2(x, mu=1), 0, 1)
Y<- rLOGNO(200)
hist(Y)
par(op)

# library(gamlss)
# data(abdom)
# h1<-gamlss(y~cs(x), family=LOGNO, data=abdom)#fits the log-Normal distribution
# h2<-gamlss(y~cs(x), family=LNO, data=abdom)  #should be identical to the one above
# to change to square root transformation, i.e. fix nu=0.5
# h3<-gamlss(y~cs(x), family=LNO, data=abdom, nu.fix=TRUE, nu.start=0.5)
```

---

LO *Logistic distribution for fitting a GAMLSS*

---

## Description

The function `LO()`, or equivalently `Logistic()`, defines the logistic distribution, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function gamlss()

## Usage

```
LO(mu.link = "identity", sigma.link = "log")
dLO(x, mu = 0, sigma = 1, log = FALSE)
pLO(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLO(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rLO(n, mu = 0, sigma = 1)
```

## Arguments

| | |
|---|---|
| `mu.link` | Defines the `mu.link`, with "identity" link as the default for the mu parameter |
| `sigma.link` | Defines the `sigma.link`, with "log" link as the default for the sigma parameter |
| `x`, `q` | vector of quantiles |
| `mu` | vector of location parameter values |
| `sigma` | vector of scale parameter values |
| `log`, `log.p` | logical; if TRUE, probabilities p are given as log(p). |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

| | |
|---|---|
| p | vector of probabilities. |
| n | number of observations. If `length(n) > 1`, the length is taken to be the number required |

## Details

Definition file for Logistic distribution.

$$f(y|\mu,\sigma) = \frac{1}{\sigma} e^{-\left(\frac{y-\mu}{\sigma}\right)} [1 + e^{-\left(\frac{y-\mu}{\sigma}\right)}]^{-2}$$

for $y = (-\infty, \infty)$, $\mu = (-\infty, \infty)$ and $\sigma > 0$, see page 368 of Rigby et al. (2019).

## Value

`LO()` returns a `gamlss.family` object which can be used to fit a logistic distribution in the `gamlss()` function. `dLO()` gives the density, `pLO()` gives the distribution function, `qLO()` gives the quantile function, and `rLO()` generates random deviates for the logistic distribution. The latest functions are based on the equivalent `R` functions for logistic distribution.

## Note

$\mu$ is the mean and $\sigma\pi/\sqrt{3}$ is the standard deviation for the logistic distribution

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

`gamlss.family`, `NO`, `TF`

## Examples

```
LO()# gives information about the default links for the Logistic distribution
plot(function(y) dLO(y, mu=10 ,sigma=2), 0, 20)
plot(function(y) pLO(y, mu=10 ,sigma=2), 0, 20)
plot(function(y) qLO(y, mu=10 ,sigma=2), 0, 1)
# library(gamlss)
# data(abdom)
# h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=LO, data=abdom) # fits
# plot(h)
```

---

LOGITNO *Logit Normal distribution for fitting in GAMLSS*

---

## Description

The functions dLOGITNO, pLOGITNO, qLOGITNO and rLOGITNO define the density, distribution function, quantile function and random generation for the logit-normal distribution. The function LOGITNO can be used for fitting the distribution in gamlss().

## Usage

```
LOGITNO(mu.link = "logit", sigma.link = "log")
dLOGITNO(x, mu = 0.5, sigma = 1, log = FALSE)
pLOGITNO(q, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLOGITNO(p, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rLOGITNO(n, mu = 0.5, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | the link function for mu |
| sigma.link | the link function for sigma |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function in LOGITNO is defined as

$$f(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}y(1-y)} \exp\left(-\frac{1}{2\sigma^2}\left[\log(y/(1-y) - \log(\mu/(1-\mu)\right]^2\right)$$

for $0 < y < 1$, $0 < \mu < 1$ and $\sigma > 0$ see p 463 of Rigby et al. (2019).

## Value

LOGITNO() returns a gamlss.family object which can be used to fit a logit-normal distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, LOGNO

## Examples

```
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dLOGITNO(x), 0, 1)
curve(pLOGITNO(x), 0, 1)
curve(qLOGITNO(x), 0, 1)
Y<- rLOGITNO(200)
hist(Y)
par(op)

# plotting the d, p, q, and r functions
# sigma 3
op<-par(mfrow=c(2,2))
curve(dLOGITNO(x, sigma=3), 0, 1)
curve(pLOGITNO(x, sigma=3), 0, 1)
curve(qLOGITNO(x, sigma=3), 0, 1)
Y<- rLOGITNO(200, sigma=3)
hist(Y)
par(op)
```

LQNO *Normal distribution with a specific mean and variance relationship for fitting a GAMLSS model*

## Description

The function LQNO() defines a normal distribution family, which has a specific mean and variance relationship. The distribution can be used in a GAMLSS fitting using the function gamlss(). The mean of LQNO is equal to mu. The variance is equal to mu*(1+sigma*mu) so the standard deviation is sqrt(mu*(1+sigma*mu)). The function is found useful in modelling small RNA sequencing experiments see Argyropoulos *et al.* (2017. The functions dLQNO, pLQNO, qLQNO and rLQNO define the density, distribution function, quantile function (inverse cdf) and random generation for the LQNO() parametrization of the normal distribution.

## Usage

```
LQNO(mu.link = "log", sigma.link = "log")
dLQNO(x, mu = 1, sigma = 1, log = FALSE)
pLQNO(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLQNO(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rLQNO(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | mu link function with "log" as default |
| sigma.link | mu link function with "log" as default |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x]. |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

LQNO stands for Linear Quadratic Normal Family, in which the variance is a linear quadratic function of the mean: Var(Y) = mu*(1+sigma*mu). This is created to facilitate the analysis of data coming from small RNA sequencing experiments, basically counts of short RNAs that one isolates from cells or biofluids such as urine, plasma or cerebrospinal fluid. Argyropoulos *et al.* (2017) showing that the LQNO distribution (and the Negative Binomial which implements the same mean- variance relationship) are highly accurate approximations to the generative models of the signals in these experiments

## Value

The function LQNO returns a gamlss.family object which can be used to fit this specific form of the normal distribution family in the gamlss() function.

## Note

The mu parameters must be positive so for the relationship Var(Y) = mu*(1+sigma*mu) to be valid.

## Author(s)

Christos Argyropoulos

## References

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Argyropoulos C, Etheridge A, Sakhanenko N, Galas D. (2017) Modeling bias and variation in the stochastic processes of small RNA sequencing. *Nucleic Acids Res.* 2017 Mar 27. doi: 10.1093/nar/gkx199. [Epub ahead of print] PubMed PMID: 28369495.

## See Also

NO,NO2, NOF

## Examples

```
LQNO()# gives information about the default links for the normal distribution
# a comparison of different Normal models
#m1 <- gamlss(y~pb(x), sigma.fo=~pb(x), data=abdom,  family=NO(mu.link="log"))
#m2 <- gamlss(y~pb(x), sigma.fo=~pb(x), data=abdom, family=LQNO)
#m3 <- gamlss(y~pb(x), sigma.fo=~pb(x), data=abdom, family=NOF(mu.link="log"))
#AIC(m1,m2,m3)
```

---

# make.link.gamlss *Create a Link for GAMLSS families*

---

## Description

The function make.link.gamlss() is used with gamlss.family distributions in package **gamlss()**. Given a link, it returns a link function, an inverse link function, the derivative dpar/deta where 'par' is the appropriate distribution parameter and a function for checking the domain. It differs from the usual make.link of glm() by having extra links as the logshifto1, and the own. For the use of the own link see the example bellow. show.link provides a way in which the user can identify the link functions available for each gamlss distribution. If your required link function is not available for any of the gamlss distributions you can add it in.

## Usage

```
make.link.gamlss(link)
show.link(family = "NO")
```

## Arguments

| | |
|---|---|
| link | character or numeric; one of "logit", "probit", "cloglog", "identity", "log", "sqrt", "1/mu^2", "inverse", "logshifted", "logitshifted", or number, say lambda resulting in power link $\mu^{\lambda}$. |
| family | a gamlss distribution family |

## Details

The own link function is added to allow the user greater flexibility. In order to used the own link function for any of the parameters of the distribution the own link should appear in the available links for this parameter. You can check this using the function show.link. If the own do not appear in the list you can create a new function for the distribution in which own is added in the list. For example the first line of the code of the binomial distribution, BI, has change from

"mstats <- checklink("mu.link", "Binomial", substitute(mu.link), c("logit", "probit", "cloglog", "log")), in version 1.0-0 of gamlss, to

"mstats <- checklink("mu.link", "Binomial", substitute(mu.link), c("logit", "probit", "cloglog", "log", "own"))

in version 1.0-1. Given that the parameter has own as an option the user needs also to define the following four new functions in order to used an own link.

i) own.linkfun

ii) own.linkinv

iii) own.mu.eta and

iv) own.valideta.

An example is given below.

Only one parameter of the distribution at a time is allowed to have its own link, (unless the same four own functions above are suitable for more that one parameter of the distribution).

Note that from **gamlss** version 1.9-0 the user can introduce its own link function by define an appropriate function, (see the example below).

## Value

For the make.link.gamlss a list with components

linkfun: Link function function(parameter)

linkinv: Inverse link function function(eta)

mu.eta: Derivative function(eta) dparameter/deta

valideta: function(eta) TRUE if all of eta is in the domain of linkinv.

For the show.link a list with components the available links for the distribution parameters

## Note

For the links involving parameters as in `logshifted` and `logitshifted` the parameters can be passed in the definition of the distribution by calling the `checklink` function, for example in the definition of the `tau` parameter in BCPE distribution the following call is made: `tstats <- checklink( "tau.link", "Box Cox Power Exponential", substitute(tau.link), c("logshifted", "log", "identity"), par.link = c(1))`

## Author(s)

Mikis Stasinopoulos and Bob Rigby

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
str(make.link.gamlss("logshiftto1"))
l2<-make.link.gamlss("logshiftto1")
l2$linkfun(2) # should close to zero (Note that 0.00001 is added)
l2$linkfun(1-0.00001) # should be -Inf but it is large negative
#-------------------------------------------------------
# now use the own link function
# first if the distribution allows you
show.link(BI)
# seems OK now define the four own functions
# First try the probit link using the own link function
# 1: the linkfun function
own.linkfun <- function(mu) { qNO(p=mu)}
# 2: the inverse link function
own.linkinv <- function(eta) {
              thresh <- -qNO(.Machine$double.eps)
               eta <- pmin(thresh, pmax(eta, -thresh))
              pNO(eta)}
# 3: the dmu/deta function
own.mu.eta <- function(eta) pmax(dNO(eta), .Machine$double.eps)
```

```
# 4: the valideta function
own.valideta <- function(eta) TRUE

## bring the data
# library(gamlss)
#data(aep)
# fitting the model using "own"
# h1<-gamlss(y~ward+loglos+year, family=BI(mu.link="own"), data=aep)
# model h1 should be identical to the probit
# h2<-gamlss(y~ward+loglos+year, family=BI(mu.link="probit"), data=aep)
# now using a function instead of "own"
probittest <- function()
{
linkfun <- function(mu) { qNO(p=mu)}
linkinv <- function(eta)
            {
              thresh <- -qNO(.Machine$double.eps)
               eta <- pmin(thresh, pmax(eta, -thresh))
              pNO(eta)
            }
mu.eta <- function(eta) pmax(dNO(eta), .Machine$double.eps)
valideta <- function(eta) TRUE
link <- "probitTest"
structure(list(linkfun = linkfun, linkinv = linkinv, mu.eta = mu.eta,
        valideta = valideta, name = link), class = "link-gamlss")
}
# h3<-gamlss(y~ward+loglos+year, family=BI(mu.link=probittest()), data=aep)
# Second try the complementary log-log
# using the Gumbel distribution
own.linkfun <- function(mu) { qGU(p=mu)}
own.linkinv <- function(eta) {
              thresh <- -qGU(.Machine$double.eps)
               eta <- pmin(thresh, pmax(eta, -thresh))
              pGU(eta)}
own.mu.eta <- function(eta) pmax(dGU(eta), .Machine$double.eps)
own.valideta <- function(eta) TRUE
# h1 and h2 should be identical to cloglog
# h1<-gamlss(y~ward+loglos+year, family=BI(mu.link="own"), data=aep)
# h2<-gamlss(y~ward+loglos+year, family=BI(mu.link="cloglog"), data=aep)
# note that the Gumbel distribution is negatively skew
# for a positively skew link function we can used the Reverse Gumbel
revloglog  <- function()
{
linkfun <- function(mu) { qRG(p=mu)}
linkinv <- function(eta) {
              thresh <- -qRG(.Machine$double.eps)
               eta <- pmin(thresh, pmax(eta, -thresh))
              pRG(eta)}
mu.eta <- function(eta) pmax(dRG(eta), .Machine$double.eps)
valideta <- function(eta) TRUE
link <- "revloglog"
structure(list(linkfun = linkfun, linkinv = linkinv, mu.eta = mu.eta,
        valideta = valideta, name = link), class = "link-gamlss")
```

```
}
# h1<-gamlss(y~ward+loglos+year, family=BI(mu.link=revloglog()), data=aep)
# a considerable improvement in the deviance
# try a shifted logit link function from -1, 1
own.linkfun <- function(mu)
             { shift = c(-1,1)
               log((mu-shift[1])/(shift[2]-mu))
             }
own.linkinv <- function(eta)
            {
            shift = c(-1,1)
            thresh <- -log(.Machine$double.eps)
               eta <- pmin(thresh, pmax(eta, -thresh))
                      shift[2]-(shift[2]-shift[1])/(1 + exp(eta))
            }
own.mu.eta <- function(eta)
            {
        shift = c(-1,1)
            thresh <- -log(.Machine$double.eps)
               res <- rep(.Machine$double.eps, length(eta))
            res[abs(eta) < thresh] <- ((shift[2]-shift[1])*exp(eta)/(1 +
                                exp(eta))^2)[abs(eta) < thresh]
            res
            }
own.valideta <- function(eta) TRUE
#----------
str(make.link.gamlss("own"))
l2<-make.link.gamlss("own")
l2$linkfun(0) # should be zero
l2$linkfun(1) # should be Inf
l2$linkinv(-5:5)
```

---

MN3 *Multinomial distribution in GAMLSS*

---

## Description

The set of function presented here is useful for fitting multinomial regression within gamlss.

## Usage

```
MN3(mu.link = "log", sigma.link = "log")
MN4(mu.link = "log", sigma.link = "log", nu.link = "log")
MN5(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
MULTIN(type = "3")
fittedMN(model)

dMN3(x, mu = 1, sigma = 1, log = FALSE)
dMN4(x, mu = 1, sigma = 1, nu = 1, log = FALSE)
dMN5(x, mu = 1, sigma = 1, nu = 1, tau = 1, log = FALSE)
```

```
pMN3(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
pMN4(q, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
pMN5(q, mu = 1, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE)

qMN3(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qMN4(p, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qMN5(p, mu = 1, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE)

rMN3(n, mu = 1, sigma = 1)
rMN4(n, mu = 1, sigma = 1, nu = 1)
rMN5(n, mu = 1, sigma = 1, nu = 1, tau = 1)
```

## Arguments

| | |
|---|---|
| `mu.link` | the link function for mu |
| `sigma.link` | the link function for sigma |
| `nu.link` | the link function for nu |
| `tau.link` | the link function for tau |
| `x` | the x variable |
| `q` | vector of quantiles |
| `p` | vector of probabilities |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x] otherwise, P[X > x]. |
| `log.p` | logical; if TRUE, probabilities p are given as log(p). |
| `log` | logical; if TRUE, probabilities p are given as log(p). |
| `n` | the number of observations |
| `mu` | the mu parameter |
| `sigma` | the sigma parameter |
| `nu` | the nu parameter |
| `tau` | the tau parameter |
| `type` | permitted values are 2 (Binomial), 3, 4, and 5 |
| `model` | a gamlss multinomial fitted model |

## Details

GAMLSS is in general not suitable for multinomial regression. Nevertheless multinomial regression can be fitted within GAMLSS if the response variable y has less than five categories. The function here provide the facilities to do so. The functions `MN3()`, `MN4()` and `MN5()` fit multinomial responses with 3, 4 and 5 categories respectively. The function `MULTIN()` can be used instead of `MN3()`, `MN4()` and `MN5()` by specifying the number of levels of the response. Note that `MULTIN(2)` will produce a binomial fit.

## Value

returns a `gamlss.family` object which can be used to fit a binomial distribution in the `gamlss()` function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Vlasios Voudouris

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BI

## Examples

```
dMN3(3)
pMN3(2)
qMN3(.6)
rMN3(10)
```

---

momentSK *Sample and theoretical Moment and Centile Skewness and Kurtosis Functions*

---

## Description

The functions momentSK(), centileSK(), centileSkew() and centileKurt(), calculate sample statistics related to skewness and kurtosis. The function theoCentileSK() calculates the theoretical centile statistics from a given gamlss.family distribution. The plotCentileSK() plots the theoretical centile skewness and kurtosis against p (see below).

The function checkMomentSK() can be use to check (a) whether the moment skewness and kurtosis of a fitted model are modelled adequantly (the residuals of the model are used). (b) whether a given sample display skewness or kurtosis.

## Usage

```
momentSK(x,  weights=NULL)
centileSK(x, cent = c(1, 25), weights=NULL)
centileSkew(x, cent = 1, weights=NULL)
centileKurt(x, cent = 1, weights=NULL)

theoCentileSK(fam = "NO", p = 0.01, ...)
plotCentileSK(fam = "NO", plotting = c("skew", "kurt", "standKurt"),
              add = FALSE, col = 1, lty = 1, lwd = 1, ylim = NULL, ...)

checkMomentSK(x, weights=NULL, add = FALSE, bootstrap = TRUE, no.bootstrap = 99,
               col.bootstrap = "lightblue", pch.bootstrap = 21,
               asCharacter = TRUE, col.point = "black", pch.point = 4,
               lwd.point = 2,  text.to.show = NULL, cex.text = 1.5,
               col.text = "black", show.legend = TRUE)

checkCentileSK(x,weights=NULL, type = c("central", "tail"), add = FALSE,
              bootstrap = TRUE, no.bootstrap = 99,
              col.bootstrap = "lightblue", pch.bootstrap = 21,
              asCharacter = TRUE, col.point = "black", pch.point = 4,
              lwd.point = 2,  text.to.show = NULL, cex.text = 1.5,
              col.text = "black", show.legend = TRUE)
```

## Arguments

| | |
|---|---|
| x | data vector or gamlss model |
| weights | prior weights for the x |
| cent | the centile required |
| type | For centile skewness and kurtosis only whether "central" (default) or "tail") |
| fam | A gamlss distribution family |
| plotting | what to plot |
| add | whether to add the line to the existing plot |
| col | the colour of the line |
| lty | the type of the line |
| lwd | the width of the line |
| ylim | the y limit of the graph |
| p | the value determiming the centile skewness or kurtosis |
| ... | additional arguments pass to `theoCentileSK()` function i.e. the values of the distribution parameters |
| bootstrap | whether a plot of the bootstrap skewness and kurtosis measures should be added in the plot |
| no.bootstrap | the number of boostrap skewness and kurtosis measures |
| col.bootstrap | the coloue for boostraps |

| | |
|---|---|
| pch.bootstrap | the point type of boostraps |
| asCharacter | whether to plot the estimated skewness and kurtosis measure as character or as point |
| col.point | the colour of the skewness and kurtosis measure |
| pch.point | the point type of the skewness and kurtosis measure |
| lwd.point | the width of the plotted point |
| text.to.show | to display text different from variable or model |
| cex.text | the size of the text |
| col.text | the colour of the text |
| show.legend | whether to show the legent |

## Details

Those function calculate sample moment and centile skewness and kurtosis statistics and theoretical centile values for a specific distribution.

## Value

Different functions produce different output: The function `momentSK()` produce:

| | |
|---|---|
| mom.skew: | sample moment skewness |
| trans.mom.skew: | sample transformed moment skewness |
| mom.kurt: | sample moment kurtosis |
| excess.mom.kurt: | sample excess moment kurtosis |
| trans.mom.kurt: | sample ransformed moment excess kurtosis |
| jarque.bera.test: | the value of the Jarque-bera test for testing whether skewness and excess kurtosis are zero or not |

The function `centileSK()` produces:

| | |
|---|---|
| S0.25: | sample centile central skewness |
| S0.01: | sample centile tail skewness |
| K0.01: | sample centile kurtosis |
| standK0.01: | standardised centile kurtosis, (K0.01/3.449) |
| exc.K0.01: | excess centile kurtosis, (K0.01-3.449) |
| trans.K0.01: | transfored excess centile kurtosis, (exc.K0.01/(1+abs(exc.K0.01)) |

The function `centileSkew()` for a given argument `p` produces:

| | |
|---|---|
| p: | the value determiming the centile skewness |
| Sp: | sample centile skewness at p |

The function `centileKurt()` for a given argument `p` produces:

| | |
|---|---|
| `p` | the value determiming the centile kurtosis |
| `Kp` | sample centile kurtosis at `p` |
| `sKp` | sample standardised centile kurtosis at `p` |
| `ex.Kp:` | sample excess centile kurtosis at `p` |
| `teKp:` | sample transformed excess centile kurtosis at `p` |

The function `theoCentileSK` for a given `gamlss.family` produces:

| | |
|---|---|
| `IR` | the interquartile range of the distribution |
| `SIR` | the semi interquartile range of the distribution |
| `S_0.25` | the central skewness of the distribution |
| `S_0.01:` | the tail skewness of the distribution |
| `K_0.01:` | the centile kurtosis of the distribution |
| `sK_0.01:` | the standardised centile kurtosis of the distribution |

## Author(s)

Mikis Stasinopoulos, Bobert Rigby, Gillain Heller and Fernanda De Bastiani.

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

`gamlss.family`

## Examples

```
Y <- rSEP3(1000)
momentSK(Y)
centileSK(Y)
centileSkew(Y, cent=20)
centileKurt(Y, cent=30)

theoCentileSK("BCCG", mu=2, sigma=.2, nu=2)
```

```
plotCentileSK(fam="BCCG",  mu=2, sigma=.2, nu=2)

checkMomentSK(Y)
checkCentileSK(Y)
checkCentileSK(Y, type="tail")
```

---

NBF *Negative Binomial Family distribution for fitting a GAMLSS*

---

## Description

The NBF() function defines the Negative Binomial family distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dNBF, pNBF, qNBF and rNBF define the density, distribution function, quantile function and random generation for the negative binomial family, NBF(), distribution.

The functions dZINBF, pZINBF, qZINBF and rZINBF define the density, distribution function, quantile function and random generation for the zero inflated negative binomial family, ZINBF(), distribution a four parameter distribution.

## Usage

```
NBF(mu.link = "log", sigma.link = "log", nu.link = "log")

dNBF(x, mu = 1, sigma = 1, nu = 2, log = FALSE)

pNBF(q, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)

qNBF(p, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)

rNBF(n, mu = 1, sigma = 1, nu = 2)

ZINBF(mu.link = "log", sigma.link = "log", nu.link = "log",
      tau.link = "logit")

dZINBF(x, mu = 1, sigma = 1, nu = 2, tau = 0.1, log = FALSE)

pZINBF(q, mu = 1, sigma = 1, nu = 2, tau = 0.1, lower.tail = TRUE,
      log.p = FALSE)

qZINBF(p, mu = 1, sigma = 1, nu = 2, tau = 0.1, lower.tail = TRUE,
      log.p = FALSE)

rZINBF(n, mu = 1, sigma = 1, nu = 2, tau = 0.1)
```

## Arguments

| | |
|---|---|
| mu.link | The link function for mu |
| sigma.link | The link function for sigma |
| nu.link | The link function for nu |
| tau.link | The link function for tau |
| x | vector of (non-negative integer) |
| mu | vector of positive means |
| sigma | vector of positive dispersion parameter |
| nu | vector of power parameter |
| tau | vector of inflation parameter |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |

## Details

The definition for Negative Binomial Family distribution , NBF, is similar to the Negative Binomial type I. The probability function of the NBF can be obtained by replacing $\sigma$ with $\sigma\mu^{\nu-2}$ where $\nu$ is a power parameter. The distribution has mean $\mu$ and variance $\mu + \sigma\mu^{\nu}$. For more details see pp 507-508 of Rigby *et al.* (2019).

The zero inflated negative binomial family ZINBF is defined as an inflated at zero NBF.

## Value

returns a gamlss.family object which can be used to fit a Negative Binomial Family distribution in the gamlss() function.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Anscombe, F. J. (1950) Sampling theory of the negative binomial and logarithmic distributions, *Biometrika*, **37**, 358-382.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

NBI, NBII

## Examples

```
NBF() # default link functions for the Negative Binomial Family
# plotting the distribution
plot(function(y) dNBF(y, mu = 10, sigma = 0.5, nu=2 ), from=0,
     to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rNBF(1000, mu=5, sigma=0.5, nu=2))
r <- barplot(tN, col='lightblue')
# zero inflated NBF
ZINBF() # default link functions  for the zero inflated NBF
# plotting the distribution
plot(function(y) dZINBF(y, mu = 10, sigma = 0.5, nu=2, tau=.1 ),
     from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rZINBF(1000, mu=5, sigma=0.5, nu=2, tau=0.1))
r <- barplot(tN, col='lightblue')
## Not run:
library(gamlss)
data(species)
species <- transform(species, x=log(lake))
m6 <- gamlss(fish~poly(x,2), sigma.fo=~1, data=species, family=NBF,
          n.cyc=200)
fitted(m6, "nu")[1]

## End(Not run)
```

---

NBI *Negative Binomial type I distribution for fitting a GAMLSS*

---

## Description

The NBI() function defines the Negative Binomial type I distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dNBI, pNBI, qNBI and rNBI define the density, distribution function, quantile function and random generation for the Negative Binomial type I, NBI(), distribution.

## Usage

```
NBI(mu.link = "log", sigma.link = "log")
dNBI(x, mu = 1, sigma = 1, log = FALSE)
pNBI(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNBI(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNBI(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive means |
| sigma | vector of positive despersion parameter |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

Definition file for Negative Binomial type I distribution.

$$P(Y=y|\mu,\sigma)=\frac{\Gamma(y+\frac{1}{\sigma})}{\Gamma(\frac{1}{\sigma})\Gamma(y+1)}\ \left(\frac{\sigma\mu}{1+\sigma\mu}\right)^{y}\ \left(\frac{1}{1+\sigma\mu}\right)^{1/\sigma}$$

for $y=0,1,2,\ldots,\infty$, $\mu>0$ and $\sigma>0$. This parameterization is equivalent to that used by Anscombe (1950) except he used $\alpha=1/\sigma$ instead of $\sigma$, see also pp. 483-485 of Rigby *et al.* (2019). The expected value is $E(y)=\mu$ and the variance $E(y)=\mu+\sigma\mu^2$.

## Value

returns a gamlss.family object which can be used to fit a Negative Binomial type I distribution in the gamlss() function.

## Warning

For values of $\sigma<0.0001$ the d,p,q,r functions switch to the Poisson distribution

## Note

$\mu$ is the mean and $(\mu+\sigma\mu^2)^{0.5}$ is the standard deviation of the Negative Binomial type I distribution (so $\sigma$ is the dispersion parameter in the usual GLM for the negative binomial type I distribution)

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Anscombe, F. J. (1950) Sampling theory of the negative bimomial and logarithmic distributiona, *Biometrika*, **37**, 358-382.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NBII, PIG, SI

## Examples

```
NBI()  # gives information about the default links for the Negative Binomial type I distribution
# plotting the distribution
plot(function(y) dNBI(y, mu = 10, sigma = 0.5 ), from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rNBI(1000, mu=5, sigma=0.5))
r <- barplot(tN, col='lightblue')
# library(gamlss)
# data(aids)
# h<-gamlss(y~cs(x,df=7)+qrt, family=NBI, data=aids) # fits the model
# plot(h)
# pdf.plot(family=NBI, mu=10, sigma=0.5, min=0, max=40, step=1)
```

---

# NBII *Negative Binomial type II distribution for fitting a GAMLSS*

---

## Description

The NBII() function defines the Negative Binomial type II distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dNBII, pNBII, qNBII and rNBII define the density, distribution function, quantile function and random generation for the Negative Binomial type II, NBII(), distribution.

## Usage

```
NBII(mu.link = "log", sigma.link = "log")
dNBII(x, mu = 1, sigma = 1, log = FALSE)
pNBII(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNBII(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNBII(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive means |
| sigma | vector of positive despersion parameter |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

Definition file for Negative Binomial type II distribution.

$$P(Y=y|\mu,\sigma)=\frac{\Gamma(y+\frac{\mu}{\sigma})\sigma^y}{\Gamma(\frac{\mu}{\sigma})\Gamma(y+1)(1+\sigma)^{y+\mu/\sigma}}$$

for $y=0,1,2,...,\infty$, $\mu>0$ and $\sigma>0$. This parameterization was used by Evans (1953) and also by Johnson *et al.* (1993) p 200, see also pp. 485-487 of Rigby *et al.* (2019). Note that in this parameterization $E(y)=\mu$ and $Var(y)=\mu(1+\sigma\mu)$.

## Value

returns a gamlss.family object which can be used to fit a Negative Binomial type II distribution in the gamlss() function.

## Note

$\mu$ is the mean and $[(1+\sigma)\mu]^{0.5}$ is the standard deviation of the Negative Binomial type II distribution, so $\sigma$ is a dispersion parameter

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Evans, D. A. (1953). Experimental evidence concerning contagious distributions in ecology. *Biometrika*, **40**: 186-211.

Johnson, N. L., Kotz, S. and Kemp, A. W. (1993). *Univariate Discrete Distributions*, 2nd edn. Wiley, New York.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NBI, PIG, SI

## Examples

```
NBII()  # gives information about the default links for the Negative Binomial type II distribution
# plotting the distribution
plot(function(y) dNBII(y, mu = 10, sigma = 0.5 ), from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rNBII(1000, mu=5, sigma=0.5))
r <- barplot(tN, col='lightblue')
# library(gamlss)
# data(aids)
# h<-gamlss(y~cs(x,df=7)+qrt, family=NBII, data=aids) # fits a model
# plot(h)
# pdf.plot(family=NBII, mu=10, sigma=0.5, min=0, max=40, step=1)
```

---

# NET *Normal Exponential t distribution (NET) for fitting a GAMLSS*

---

## Description

This function defines the Power Exponential t distribution (NET), a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dNET, pNET define the density and distribution function the NET distribution.

## Usage

```
NET(mu.link = "identity", sigma.link = "log", nu.link ="identity",
      tau.link = "identity")
pNET(q, mu=0, sigma=1, nu=1.5, tau=2,  lower.tail = TRUE, log.p = FALSE)
dNET(x, mu=0, sigma=1, nu=1.5, tau=2, log=FALSE)
qNET(p, mu=0, sigma=1, nu=1.5, tau=2,  lower.tail = TRUE, log.p = FALSE)
rNET(n,  mu=0, sigma=1, nu=1.5, tau=2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse", "log" and "own" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" and "own" |
| nu.link | Defines the nu.link, and because nu is fixed we use "identity" link |
| tau.link | Defines the tau.link, and because tau is fixed we use "identity" link |
| x, q | vector of quantiles |
| p | vector of probabilities |
| n | number of observations. |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of nu parameter values |
| tau | vector of tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

The NET distribution was introduced by Rigby and Stasinopoulos (1994) as a robust distribution for a response variable with heavier tails than the normal. The NET distribution is the abbreviation of the Normal Exponential Student t distribution. The NET distribution is a four parameter continuous distribution, although in the GAMLSS implementation only the two parameters, mu and sigma, of the distribution are modelled with nu and tau fixed. The distribution takes its names because it is normal up to nu, Exponential from nu to tau (hence abs(nu)<=abs(tau)) and Student-t with nu*tau-1 degrees of freedom after tau. Maximum likelihood estimator of the third and forth parameter can be obtained, using the GAMLSS functions, find.hyper or prof.dev.

For more details about the NET distribution please refer to pp. 393-396 of of Rigby et al. (2019).

## Value

NET() returns a gamlss.family object which can be used to fit a Box Cox Power Exponential distribution in the gamlss() function. dNET() gives the density, pNET() gives the distribution function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos, D. M. (1994), Robust fitting of an additive model for variance heterogeneity, *COMPSTAT : Proceedings in Computational Statistics*, editors:R. Dutter and W. Grossmann, pp 263-268, Physica, Heidelberg.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

SStasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCPE

## Examples

```
NET()   #
data(abdom)
plot(function(x)dNET(x, mu=0,sigma=1,nu=2, tau=3), -5, 5)
plot(function(x)pNET(x, mu=0,sigma=1,nu=2, tau=3), -5, 5)
# fit NET with nu=1 and tau=3
# library(gamlss)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=NET,
#        data=abdom, nu.start=2, tau.start=3)
#plot(h)
```

---

NO *Normal distribution for fitting a GAMLSS*

---

## Description

The function NO() defines the normal distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with mean equal to the parameter mu and sigma equal the standard deviation. The functions dNO, pNO, qNO and rNO define the density, distribution function, quantile function and random generation for the NO parameterization of the normal distribution. [A alternative parameterization with sigma equal to the variance is given in the function NO2()]

## Usage

```
NO(mu.link = "identity", sigma.link = "log")
dNO(x, mu = 0, sigma = 1, log = FALSE)
pNO(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNO(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNO(n, mu = 0, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parametrization of the normal distribution given in the function NO() is

$$f(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}(\frac{y-\mu}{\sigma})^2\right]$$

for $y = (-\infty, \infty)$, $\mu = (-\infty, +\infty)$ and $\sigma > 0$ see pp. 369-370 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a normal distribution in the gamlss() function.

## Note

For the function NO(), $\mu$ is the mean and $\sigma$ is the standard deviation (not the variance) of the normal distribution.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NO2

## Examples

```
NO()# gives information about the default links for the normal distribution
plot(function(y) dNO(y, mu=10 ,sigma=2), 0, 20)
plot(function(y) pNO(y, mu=10 ,sigma=2), 0, 20)
plot(function(y) qNO(y, mu=10 ,sigma=2), 0, 1)
dat<-rNO(100)
hist(dat)
# library(gamlss)
# gamlss(dat~1,family=NO) # fits a constant for mu and sigma
```

---

# NO2 *Normal distribution (with variance as sigma parameter) for fitting a GAMLSS*

---

## Description

The function NO2() defines the normal distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss() with mean equal to mu and variance equal to sigma. The functions dNO2, pNO2, qNO2 and rNO2 define the density, distribution function, quantile function and random generation for this specific parameterization of the normal distribution.

[A alternative parameterization with sigma as the standard deviation is given in the function NO()]

## Usage

```
NO2(mu.link = "identity", sigma.link = "log")
dNO2(x, mu = 0, sigma = 1, log = FALSE)
pNO2(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNO2(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNO2(n, mu = 0, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parametrization of the normal distribution given in the function NO2() is

$$f(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2}\frac{(y-\mu)^2}{\sigma}\right]$$

for $y = (-\infty, \infty)$, $\mu = (-\infty, +\infty)$ and $\sigma > 0$ see p. 370 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a normal distribution in the gamlss() function.

## Note

For the function NO(), $\mu$ is the mean and $\sigma$ is the standard deviation (not the variance) of the normal distribution. [The function NO2() defines the normal distribution with $\sigma$ as the variance.]

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NO

## Examples

```
NO()# gives information about the default links for the normal distribution
dat<-rNO(100)
hist(dat)
plot(function(y) dNO(y, mu=10 ,sigma=2), 0, 20)
plot(function(y) pNO(y, mu=10 ,sigma=2), 0, 20)
plot(function(y) qNO(y, mu=10 ,sigma=2), 0, 1)
# library(gamlss)
# gamlss(dat~1,family=NO) # fits a constant for mu and sigma
```

---

# NOF *Normal distribution family for fitting a GAMLSS*

---

## Description

The function NOF() defines a normal distribution family, which has three parameters. The distribution can be used using the function gamlss(). The mean of NOF is equal to mu. The variance is equal to sigma^2*mu^nu so the standard deviation is sigma*mu^(nu/2). The function is design for cases where the variance is proportional to a power of the mean. This is an instance of the Taylor's power low, see Enki et al. (2017). The functions dNOF, pNOF, qNOF and rNOF define the density, distribution function, quantile function and random generation for the NOF parametrization of the normal distribution family.

## Usage

```
NOF(mu.link = "identity", sigma.link = "log", nu.link = "identity")
dNOF(x, mu = 0, sigma = 1, nu = 0, log = FALSE)
pNOF(q, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)
qNOF(p, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)
rNOF(n, mu = 0, sigma = 1, nu = 0)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link with "identity" link as the default for the nu parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of power parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |

| | |
|---|---|
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parametrization of the normal distribution given in the function NOF() is

$$f(y|\mu,\sigma,\nu) = \frac{1}{\sqrt{2\pi}\sigma\mu^{\nu/2}} \exp\left[-\frac{1}{2}\frac{(y-\mu)^2}{\sigma^2\mu^{\nu}}\right]$$

for $y = (-\infty, \infty)$, $\mu = (-\infty, \infty)$, $\sigma > 0$ and $\nu = (-\infty, +\infty)$ see pp. 373-374 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a normal distribution family in the gamlss() function.

## Note

For the function NOF(), $\mu$ is the mean and $\sigma\mu^{\nu/2}$ is the standard deviation of the normal distribution family. The NOF is design for fitting regression type models where the variance is proportional to a power of the mean. Models of this type are also related to the "pseudo likelihood" models of Carroll and Rubert (1987) but here a proper likelihood is miximised.

Note that because the high correlation between the sigma and the nu parameter the mixed() method should be used in the fitting.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Davidian, M. and Carroll, R. J. (1987), Variance Function Estimation, *Journal of the American Statistical Association*, Vol. **82**, pp. 1079-1091

Enki, D G, Noufaily, A., Farrington, P., Garthwaite, P., Andrews, N. and Charlett, A. (2017) Taylor's power law and the statistical modelling of infectious disease surveillance data, Journal of the Royal Statistical Society: Series A (Statistics in Society), volume=180, number=1, pages=45-72.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NO, NO2

## Examples

```
NOF()# gives information about the default links for the normal distribution family
## Not run:
## the normal distribution, fitting a constant sigma
m1<-gamlss(y~poly(x,2), sigma.fo=~1, family=NO, data=abdom)
## the normal family, fitting a variance proportional to the mean (mu)
m2<-gamlss(y~poly(x,2), sigma.fo=~1, family=NOF, data=abdom, method=mixed(1,20))
## the nornal distribution fitting  the variance as a function of x
m3 <-gamlss(y~poly(x,2), sigma.fo=~x,   family=NO, data=abdom, method=mixed(1,20))
GAIC(m1,m2,m3)

## End(Not run)
```

---

PARETO2 *Pareto distributions for fitting in GAMLSS*

---

## Description

The functions PARETO() defines the one parameter Pareto distribution for y>1.

The functions PARETO1() defines the one parameter Pareto distribution for y>0.

The functions PARETOo1() defines the one parameter Pareto distribution for y>mu therefor requires mu to be fixed.

The functions PARETO2() and PARETO2o() define the Pareto Type 2 distribution, for y>0, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The parameters are mu and sigma in both functions but the parameterasation different. The mu is identical for both PARETO2() and PARETO2o(). The sigma in PARETO2o() is the inverse of the sigma in PARETO2() and coresponse to the usual parameter alpha of the Pareto distribution.

The functions dPARETO2, pPARETO2, qPARETO2 and rPARETO2 define the density, distribution function, quantile function and random generation for the PARETO2 parameterization of the Pareto type 2 distribution while the functions dPARETO2o, pPARETO2o, qPARETO2o and rPARETO2o define the density, distribution function, quantile function and random generation for the original PARETO2o parameterization of the Pareto type 2 distribution.

## Usage

```
PARETO(mu.link = "log")
dPARETO(x, mu = 1, log = FALSE)
pPARETO(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qPARETO(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rPARETO(n, mu = 1)

PARETO1(mu.link = "log")
dPARETO1(x, mu = 1, log = FALSE)
pPARETO1(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qPARETO1(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rPARETO1(n, mu = 1)

PARETO1o(mu.link = "log", sigma.link = "log")
dPARETO1o(x, mu = 1, sigma = 0.5, log = FALSE)
pPARETO1o(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
qPARETO1o(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
rPARETO1o(n, mu = 1, sigma = 0.5)

PARETO2(mu.link = "log", sigma.link = "log")
dPARETO2(x, mu = 1, sigma = 0.5, log = FALSE)
pPARETO2(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
qPARETO2(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
rPARETO2(n, mu = 1, sigma = 0.5)

PARETO2o(mu.link = "log", sigma.link = "log")
dPARETO2o(x, mu = 1, sigma = 0.5, log = FALSE)
pPARETO2o(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
qPARETO2o(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
rPARETO2o(n, mu = 1, sigma = 0.5)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "‘"’ link sa the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "‘log"’ as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise P[X > x] |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the one parameter Pareto distribution in the function PARETO is:

$$f(y|\mu) = \mu y^{\mu+1}$$

for $y > 1$ and $\mu > 0$.

The parameterization of the Pareto Type 1 original distribution in the function PARETO1o is:

$$f(y|\mu,\sigma) = \frac{\sigma\mu^{\sigma}}{y^{\sigma+1}}$$

for $y >= 0$, $\mu > 0$ and $\sigma > 0$ see pp. 430-431 of Rigby et al. (2019).

The parameterization of the Pareto Type 2 original distribution in the function PARETO2o is:

$$f(y|\mu,\sigma) = \frac{\sigma\mu^{\sigma}}{(y+\mu)^{\sigma+1}}$$

for $y >= 0$, $\mu > 0$ and $\sigma > 0$ see pp. 432-433 of Rigby et al. (2019).

The parameterization of the Pareto Type 2 distribution in the function PARETO2 is:

$$f(y|\mu,\sigma) = \frac{1}{\sigma}\mu^{\frac{1}{\sigma}}\,(y+\mu)^{-\frac{1}{\sigma+1}}$$

for $y >= 0$, $\mu > 0$ and $\sigma > 0$ see pp.433-434 The parameterization of the Pareto Type 1 original distribution in the function PARETO1o is:

$$f(y|\mu,\sigma) = \frac{\sigma\mu^{\sigma}}{y^{\sigma+1}}$$

for $y >= 0$, $\mu > 0$ and $\sigma > 0$ see pp. 430-431 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a Pareto type 2 distribution in the gamlss() function.

## Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos

## References

Johnson, N., Kotz, S., and Balakrishnan, N. (1997). *Discrete Multivariate Distributions.* Wiley-Interscience, NY, USA.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
par(mfrow=c(2,2))
y<-seq(0.2,20,0.2)
plot(y, dPARETO2(y), type="l" , lwd=2)
q<-seq(0,20,0.2)
plot(q, pPARETO2(q), ylim=c(0,1), type="l", lwd=2)
p<-seq(0.0001,0.999,0.05)
plot(p, qPARETO2(p), type="l", lwd=2)
dat <- rPARETO2(100)
hist(rPARETO2(100), nclass=30)
#summary(gamlss(a~1, family="PARETO2"))
```

---

PE *Power Exponential distribution for fitting a GAMLSS*

---

## Description

The functions define the Power Exponential distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dPE, pPE, qPE and rPE define the density, distribution function, quantile function and random generation for the specific parameterization of the power exponential distribution showing below. The functions dPE2, pPE2, qPE2 and rPE2 define the density, distribution function, quantile function and random generation of a standard parameterization of the power exponential distribution.

## Usage

```
PE(mu.link = "identity", sigma.link = "log", nu.link = "log")
dPE(x, mu = 0, sigma = 1, nu = 2, log = FALSE)
pPE(q, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
qPE(p, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
rPE(n, mu = 0, sigma = 1, nu = 2)
PE2(mu.link = "identity", sigma.link = "log", nu.link = "log")
dPE2(x, mu = 0, sigma = 1, nu = 2, log = FALSE)
pPE2(q, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
qPE2(p, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
rPE2(n, mu = 0, sigma = 1, nu = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of kurtosis parameter |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

Power Exponential distribution (PE) is defined as:

$$f(y|\mu,\sigma,\nu) = \frac{\nu \exp[-|z|^{\nu}]}{2c\sigma\Gamma(\frac{1}{\nu})}$$

where $z = (y-\mu)/c\sigma$ and $c^2 = \Gamma(1/\nu)\left[/\Gamma(3/\nu)\right]^{-1}$, for $y = (-\infty, +\infty)$, $\mu = (-\infty, +\infty)$, $\sigma > 0$ and $\nu > 0$. This parametrization was used by Nelson (1991) and ensures $\mu$ is the mean and $\sigma$ is the standard deviation of y (for all parameter values of $\mu$, $\sigma$ and $\nu$ within the ranges above), see p. 374 of Rigby et al. (2019)

Thw Power Exponential distribution (PE2) is defined as

$$f(y|\mu,\sigma,\nu) = \frac{\nu \exp[-|z|^{\nu}]}{2\sigma\Gamma\left(\frac{1}{\nu}\right)}$$

see p. 376 of Rigby et al. (2019)

## Value

returns a gamlss.family object which can be used to fit a Power Exponential distribution in the gamlss() function.

## Note

$\mu$ is the mean and $\sigma$ is the standard deviation of the Power Exponential distribution

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Nelson, D.B. (1991) Conditional heteroskedasticity in asset returns: a new approach. *Econometrica*, **57**, 347-370.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCPE

## Examples

```
PE()# gives information about the default links for the Power Exponential distribution
# library(gamlss)
# data(abdom)
# h1<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=PE, data=abdom) # fit
# h2<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=PE2, data=abdom) # fit
# plot(h1)
# plot(h2)
# leptokurtotic
plot(function(x) dPE(x, mu=10,sigma=2,nu=1), 0.0, 20,
 main = "The PE  density mu=10,sigma=2,nu=1")
# platykurtotic
plot(function(x) dPE(x, mu=10,sigma=2,nu=4), 0.0, 20,
 main = "The PE  density mu=10,sigma=2,nu=4")
```

---

# PIG *The Poisson-inverse Gaussian distribution for fitting a GAMLSS model*

---

## Description

The PIG() function defines the Poisson-inverse Gaussian distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The PIG2() function is a repametrization of PIG() where mu and sigma are orthogonal see Heller *et al.* (2018).

The functions dPIG, pPIG, qPIG and rPIG define the density, distribution function, quantile function and random generation for the Poisson-inverse Gaussian PIG(), distribution. Also dPIG2, pPIG2, qPIG2 and rPIG2 are the equivalent functions for PIG2()

The functions ZAPIG() and ZIPIG() are the zero adjusted (hurdle) and zero inflated versions of the Poisson-inverse Gaussian distribution, respectively. That is three parameter distributions.

The functions dZAPIG, dZIPIG, pZAPIG,pZIPIG, qZAPIG qZIPIG rZAPIG and rZIPIG define the probability, cumulative, quantile and random generation functions for the zero adjusted and zero inflated Poisson Inverse Gaussian distributions, ZAPIG(), ZIPIG(), respectively.

## Usage

```
PIG(mu.link = "log", sigma.link = "log")
dPIG(x, mu = 1, sigma = 1, log = FALSE)
pPIG(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qPIG(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE,
     max.value = 10000)
rPIG(n, mu = 1, sigma = 1, max.value = 10000)

PIG2(mu.link = "log", sigma.link = "log")
dPIG2(x, mu=1, sigma=1, log = FALSE)
pPIG2(q, mu=1, sigma=1, lower.tail = TRUE, log.p = FALSE)
qPIG2(p, mu=1, sigma=1, lower.tail = TRUE, log.p = FALSE,
     max.value = 10000)
rPIG2(n, mu=1, sigma=1)

ZIPIG(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZIPIG(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZIPIG(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZIPIG(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE,
       max.value = 10000)
rZIPIG(n, mu = 1, sigma = 1, nu = 0.3, max.value = 10000)

ZAPIG(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZAPIG(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZAPIG(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZAPIG(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE,
      max.value = 10000)
rZAPIG(n, mu = 1, sigma = 1, nu = 0.3, max.value = 10000)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the mu.link, with "logit" link as the default for the nu parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive means |
| sigma | vector of positive dispersion parameter |
| nu | vector of zero probability parameter |
| p | vector of probabilities |

| | |
|---|---|
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| max.value | a constant, set to the default value of 10000 for how far the algorithm should look for q |

## Details

The probability function of the Poisson-inverse Gaussian distribution PIG, is given by

$$f(y|\mu,\sigma) = \left(\frac{2\alpha}{\pi}\right)^{\frac{1}{2}} \frac{\mu^y e^{\frac{1}{\sigma}} K_{y-\frac{1}{2}}(\alpha)}{(\alpha\sigma)^y y!}$$

where $\alpha^2 = \frac{1}{\sigma^2} + \frac{2\mu}{\sigma}$, for $y = 0, 1, 2, ..., \infty$ where $\mu > 0$ and $\sigma > 0$ and $K_\lambda(t) = \frac{1}{2}\int_0^\infty x^{\lambda-1} \exp\{-\frac{1}{2}t(x + x^{-1})\}dx$ is the modified Bessel function of the third kind, see also pp. 487-489 of Rigby *et al.* (2019). [Note that the above parameterization was used by Dean, Lawless and Willmot(1989). It is also a special case of the Sichel distribution SI() when $\nu = -\frac{1}{2}$.] In this parameterization $E(y) = \mu$ and $Var(y) = \mu + \sigma\mu^2$.

The probability function of the Poisson-inverse Gaussian distribution PIG2, see Heller, Couturier and Heritier (2018), is given by

$$f(y|\mu,\sigma) = \left(\frac{2\sigma}{\pi}\right)^{\frac{1}{2}} \frac{\mu^y e^{\frac{1}{\alpha}} K_{y-\frac{1}{2}}(\sigma)}{(\alpha\sigma)^y y!}$$

for $y = 0, 1, 2, ..., \infty$, $\mu > 0$ and $\sigma > 0$ and $\alpha = \left[(\mu^2 + \sigma^2)^{0.5} - \mu\right]^{-1}$, $K_\lambda(t) = \frac{1}{2}\int_0^\infty x^{\lambda-1} \exp\{-\frac{1}{2}t(x + x^{-1})\}dx$ is the modified Bessel function of the third kind, see pp. 487-489 of Rigby *et al.* (2019). In this parameterization $E(y) = \mu$ and $\sigma$ is a despertion parameter.

The definition of the zero adjusted Poison inverse Gaussian distribution, ZAPIG and the the zero inflated Poison inverse Gaussian distribution, ZIPIG, are given in p. 513 and pp. 514-515 of of Rigby *et al.* (2019), respectively.

## Value

Returns a gamlss.family object which can be used to fit a Poisson-inverse Gaussian distribution in the gamlss() function.

## Author(s)

Dominique-Laurent Couturier, Mikis Stasinopoulos, Bob Rigby and Marco Enea

## References

Dean, C., Lawless, J. F. and Willmot, G. E., A mixed poisson-inverse-Gaussian regression model, *Canadian J. Statist.*, **17**, 2, pp 171-181

Heller, G. Z., Couturier, D.L. and Heritier, S. R. (2018) Beyond mean modelling: Bias due to misspecification of dispersion in Poisson-inverse Gaussian regression *Biometrical Journal*, **2**, pp 333-342.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NBI, NBII, SI, SICHEL

## Examples

```
PIG()# gives information about the default links for the  Poisson-inverse Gaussian distribution
#plot the pdf using plot
plot(function(y) dPIG(y, mu=10, sigma = 1 ), from=0, to=50, n=50+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=50),pPIG(seq(from=0,to=50), mu=10, sigma=1), type="h")   # cdf
# generate random sample
tN <- table(Ni <- rPIG(100, mu=5, sigma=1))
r <- barplot(tN, col='lightblue')
# fit a model to the data
# library(gamlss)
# gamlss(Ni~1,family=PIG)
ZIPIG()
ZAPIG()
```

---

# PO *Poisson distribution for fitting a GAMLSS model*

---

## Description

This function PO defines the Poisson distribution, an one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dPO, pPO, qPO and rPO define the density, distribution function, quantile function and random generation for the Poisson, PO(), distribution.

## Usage

```
PO(mu.link = "log")
dPO(x, mu = 1, log = FALSE)
pPO(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qPO(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rPO(n, mu = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive means |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

Definition file for Poisson distribution.

$$f(y|\mu) = \frac{e^{-\mu}\mu^{y}}{\Gamma(y+1)}$$

for $y = 0, 1, 2, ...$ and $\mu > 0$ see ee pp 476-477 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a Poisson distribution in the gamlss() function.

## Note

$\mu$ is the mean of the Poisson distribution

## Author(s)

Bob Rigby, Mikis Stasinopoulos, and Kalliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

(Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NBI, NBII, SI, SICHEL

## Examples

```
PO()# gives information about the default links for the Poisson distribution
# fitting data using PO()

# plotting the distribution
plot(function(y) dPO(y, mu=10 ), from=0, to=20, n=20+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rPO(1000, mu=5))
 r <- barplot(tN, col='lightblue')
# library(gamlss)
# data(aids)
# h<-gamlss(y~cs(x,df=7)+qrt, family=PO, data=aids) # fits the constant+x+qrt model
# plot(h)
# pdf.plot(family=PO, mu=10, min=0, max=20, step=1)
```

---

# RG *The Reverse Gumbel distribution for fitting a GAMLSS*

---

## Description

The function RG defines the reverse Gumbel distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dRG, pRG, qRG and rRG define the density, distribution function, quantile function and random generation for the specific parameterization of the reverse Gumbel distribution.

## Usage

```
RG(mu.link = "identity", sigma.link = "log")
dRG(x, mu = 0, sigma = 1, log = FALSE)
pRG(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qRG(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rRG(n, mu = 0, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. other available link is "inverse", "log" and "own" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other links are the "inverse", "identity" and "own" |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |

| | |
|---|---|
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The specific parameterization of the reverse Gumbel distribution used in RG is

$$f(y|\mu,\sigma) = \frac{1}{\sigma}\ \exp\left\{-\left(\frac{y-\mu}{\sigma}\right) - \exp\left[-\left(\frac{y-\mu)}{\sigma}\right)\right]\right\}$$

for $y = (-\infty, \infty)$, $\mu = (-\infty, +\infty)$ and $\sigma > 0$ see pp. 370-371 of Rigby et al. (2019).

## Value

RG() returns a gamlss.family object which can be used to fit a Gumbel distribution in the gamlss() function. dRG() gives the density, pGU() gives the distribution function, qRG() gives the quantile function, and rRG() generates random deviates.

## Note

The mean of the distribution is $\mu + 0.57722\sigma$ and the variance is $\pi^2\sigma^2/6$.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
plot(function(x) dRG(x, mu=0,sigma=1), -3, 6,
 main = "{Reverse Gumbel  density mu=0,sigma=1}")
RG()# gives information about the default links for the Gumbel distribution
dat<-rRG(100, mu=10, sigma=2) # generates 100 random observations
# library(gamlss)
# gamlss(dat~1,family=RG) # fits a constant for each parameter mu and sigma
```

---

RGE *Reverse generalized extreme family distribution for fitting a GAMLSS*

---

## Description

The function RGE defines the reverse generalized extreme family distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dRGE, pRGE, qRGE and rRGE define the density, distribution function, quantile function and random generation for the specific parameterization of the reverse generalized extreme distribution given in details below.

## Usage

```
RGE(mu.link = "identity", sigma.link = "log", nu.link = "log")
dRGE(x, mu = 1, sigma = 0.1, nu = 1, log = FALSE)
pRGE(q, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qRGE(p, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
rRGE(n, mu = 1, sigma = 0.1, nu = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of the shape parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

Definition file for reverse generalized extreme family distribution.

The probability density function of the generalized extreme value distribution is obtained from Johnson *et al.* (1995), Volume 2, p76, equation (22.184) [where $(\xi, \theta, \gamma) \longrightarrow (\mu, \sigma, \nu)$].

The probability density function of the reverse generalized extreme value distribution is then obtained by replacing y by -y and $\mu$ by $-\mu$.

Hence the probability density function of the reverse generalized extreme value distribution with $\nu > 0$ is given by

$$f(y|\mu,\sigma,\nu) = \frac{1}{\sigma}\left[1+\frac{\nu(y-\mu)}{\sigma}\right]^{\frac{1}{\nu}-1} S_1(y|\mu,\sigma,\nu)$$

for

$$\mu - \frac{\sigma}{\nu} < y < \infty$$

where

$$S_1(y|\mu,\sigma,\nu) = \exp\left\{-\left[1+\frac{\nu(y-\mu)}{\sigma}\right]^{\frac{1}{\nu}}\right\}$$

and where $-\infty < \mu < y + \frac{\sigma}{\nu}$, $\sigma > 0$ and $\nu > 0$. Note that only the case $\nu > 0$ is allowed here. The reverse generalized extreme value distribution is denoted as RGE($\mu, \sigma, \nu$) or as Reverse Generalized.Extreme.Family($\mu, \sigma, \nu$).

Note the the above distribution is a reparameterization of the three parameter Weibull distribution given by

$$f(y|\alpha_1,\alpha_2,\alpha_3) = \frac{\alpha_3}{\alpha_2}\left[\frac{y-\alpha_1}{\alpha_2}\right]^{\alpha_3-1} \exp\left[-\left(\frac{y-\alpha_1}{\alpha_2}\right)^{\alpha_3}\right]$$

given by setting $\alpha_1 = \mu - \sigma/\nu$, $\alpha_2 = \sigma/\nu$, $\alpha_3 = 1/\nu$.

## Value

`RGE()` returns a `gamlss.family` object which can be used to fit a reverse generalized extreme distribution in the `gamlss()` function. `dRGE()` gives the density, `pRGE()` gives the distribution function, `qRGE()` gives the quantile function, and `rRGE()` generates random deviates.

## Note

This distribution is very difficult to fit because the y values depends on the parameter values. The `RS()` and `CG()` algorithms are not appropriate for this type of problem.

## Author(s)

Bob Rigby, Mikis Stasinopoulos and Kalliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
RGE()# default links for the reverse generalized extreme family distribution
newdata<-rRGE(100,mu=0,sigma=1,nu=5) # generates 100 random observations
# library(gamlss)
# gamlss(newdata~1, family=RGE, method=mixed(5,50)) # difficult to converse
```

---

SEP *The Skew Power exponential (SEP) distribution for fitting a GAMLSS*

---

## Description

This function defines the Skew Power exponential (SEP) distribution, a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dSEP, pSEP, qSEP and rSEP define the density, distribution function, quantile function and random generation for the Skew Power exponential (SEP) distribution.

## Usage

```
SEP(mu.link = "identity", sigma.link = "log", nu.link = "identity",
    tau.link = "log")
dSEP(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pSEP(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
     log.p = FALSE)
qSEP(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
     log.p = FALSE, lower.limit = mu - 5 * sigma,
     upper.limit = mu + 5 * sigma)
rSEP(n, mu = 0, sigma = 1, nu = 0, tau = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "$1/mu^2$" and "log" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse" and "identity" |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "$1/nu^2$" and "log" |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "$1/tau^2$", and "identity |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| lower.limit | lower limit for the golden search to find quantiles from probabilities |
| upper.limit | upper limit for the golden search to find quantiles from probabilities |

## Details

The probability density function of the Skew Power exponential distribution, (SEP), is defined as

$$f(y|n,\mu,\sigma\,\nu,\tau) == \frac{z}{\sigma}\Phi(\omega)\; f_{EP}(z,0,1,\tau)$$

for $-\infty < y < \infty$, $\mu = (-\infty,+\infty)$, $\sigma > 0$, $\nu = (-\infty,+\infty)$ and $\tau > 0$. where $z = \frac{y-\mu}{\sigma}$, $\omega = sign(z)|z|^{\tau/2}\nu\sqrt{2/\tau}$ and $f_{EP}(z,0,1,\tau)$ is the pdf of an Exponential Power distribution.

## Value

SEP() returns a gamlss.family object which can be used to fit the SEP distribution in the gamlss() function. dSEP() gives the density, pSEP() gives the distribution function, qSEP() gives the quantile function, and rSEP() generates random deviates.

## Warning

The qSEP and rSEP are slow since they are relying on golden section for finding the quantiles

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Diciccio, T. J. and Mondi A. C. (2004). Inferential Aspects of the Skew Exponential Power distribution., *JASA*, **99**, 439-450.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSU, BCT

## Examples

```
SEP()   #
plot(function(x)dSEP(x, mu=0,sigma=1, nu=1, tau=2), -5, 5,
 main = "The SEP  density mu=0,sigma=1,nu=1, tau=2")
plot(function(x) pSEP(x, mu=0,sigma=1,nu=1, tau=2), -5, 5,
 main = "The BCPE  cdf mu=0, sigma=1, nu=1, tau=2")
dat <- rSEP(100,mu=10,sigma=1,nu=-1,tau=1.5)
# library(gamlss)
# gamlss(dat~1,family=SEP, control=gamlss.control(n.cyc=30))
```

---

# SEP1 *The Skew exponential power type 1-4 distribution for fitting a GAMLSS*

---

## Description

These functions define the Skew Power exponential type 1 to 4 distributions. All of them are four parameter distributions and can be used to fit a GAMLSS model. The functions dSEP1, dSEP2, dSEP3 and dSEP4 define the probability distribution functions, the functions pSEP1, pSEP2, pSEP3 and pSEP4 define the cumulative distribution functions the functions qSEP1, qSEP2, qSEP3 and qSEP4 define the inverse cumulative distribution functions and the functions rSEP1, rSEP2, rSEP3 and rSEP4 define the random generation for the Skew exponential power distributions.

## Usage

```
SEP1(mu.link = "identity", sigma.link = "log", nu.link = "identity",
     tau.link = "log")
dSEP1(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pSEP1(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
     log.p = FALSE)
qSEP1(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
     log.p = FALSE)
rSEP1(n, mu = 0, sigma = 1, nu = 0, tau = 2)

SEP2(mu.link = "identity", sigma.link = "log", nu.link = "identity",
      tau.link = "log")
dSEP2(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pSEP2(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
      log.p = FALSE)
qSEP2(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
      log.p = FALSE)
rSEP2(n, mu = 0, sigma = 1, nu = 0, tau = 2)

SEP3(mu.link = "identity", sigma.link = "log", nu.link = "log",
      tau.link = "log")
dSEP3(x, mu = 0, sigma = 1, nu = 2, tau = 2, log = FALSE)
pSEP3(q, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
      log.p = FALSE)
qSEP3(p, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
      log.p = FALSE)

SEP4(mu.link = "identity", sigma.link = "log", nu.link = "log",
      tau.link = "log")
dSEP4(x, mu = 0, sigma = 1, nu = 2, tau = 2, log = FALSE)
pSEP4(q, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
      log.p = FALSE)
qSEP4(p, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
      log.p = FALSE)
rSEP4(n, mu = 0, sigma = 1, nu = 2, tau = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse" and "log" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse" and "identity" |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter. Other links are "identity" and "inverse" |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "inverse", and "identity |
| x, q | vector of quantiles |

| | |
|---|---|
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the Skew Power exponential distribution type 1, (SEP1), is defined as

$$f_Y(y|\mu,\sigma\,\nu,\tau) = \frac{2}{\sigma} f_{Z_1}(z) F_{Z_1}(\nu z)$$

for $-\infty < y < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$, $-\infty < \nu < \infty$, and $\tau > 0$ where $z = (y-\mu)/\sigma$, and $Z_1 \sim \texttt{PE2}(0, \tau^{1/\tau}, \tau)$, see pp 401-402 of Rigby et al. (2019).

The probability density function of the Skew Power exponential distribution type 2, (SEP2), is defined as

$$f_Y(y|\mu,\sigma\,\nu,\tau) = \frac{2}{\sigma} f_{Z_1}(z) \Phi_{Z_1}(\omega)$$

for $-\infty < y < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$, $-\infty < \nu < \infty$, and $\tau > 0$ where $z = (y-\mu)/\sigma$, and $\omega = \texttt{sign}(z)|z|^{\tau/2}\nu\sqrt{2/\tau}$, and $Z_1 \sim \texttt{PE2}(0, \tau^{1/\tau}, \tau)$, see pp 402-404 of Rigby et al. (2019).

For SEP3 and SEP3 see pp 404-406 and pp 407-408 of Rigby et al. (2019), respectively.

## Value

SEP2() returns a gamlss.family object which can be used to fit the SEP2 distribution in the gamlss() function. dSEP2() gives the density, pSEP2() gives the distribution function, qSEP2() gives the quantile function, and rSEP2() generates random deviates.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Fernadez C., Osiewalski J. and Steel M.F.J.(1995) Modelling and inference with v-spherical distributions. *JASA*, **90**, pp 1331-1340.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, SEP

## Examples

```
SEP1()
curve(dSEP4(x, mu=5 ,sigma=1, nu=2, tau=1.5), -2, 10,
          main = "The SEP4  density mu=5 ,sigma=1, nu=1, tau=1.5")
# library(gamlss)
#y<- rSEP4(100, mu=5, sigma=1, nu=2, tau=1.5);hist(y)
#m1<-gamlss(y~1, family=SEP1, n.cyc=50)
#m2<-gamlss(y~1, family=SEP2, n.cyc=50)
#m3<-gamlss(y~1, family=SEP3, n.cyc=50)
#m4<-gamlss(y~1, family=SEP4, n.cyc=50)
#GAIC(m1,m2,m3,m4)
```

---

SHASH *The Sinh-Arcsinh (SHASH) distribution for fitting a GAMLSS*

---

## Description

The Sinh-Arcsinh (SHASH) distribution is a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dSHASH, pSHASH, qSHASH and rSHASH define the density, distribution function, quantile function and random generation for the Sinh-Arcsinh (SHASH) distribution.

There are 3 different SHASH distributions implemented in GAMLSS SHASH, SHASHo and SHASH2o2 (see Details) but also there is an exponential version LOGSHASH for responses in $(0, \infty)$.

## Usage

```
SHASH(mu.link = "identity", sigma.link = "log", nu.link = "log",
      tau.link = "log")
dSHASH(x, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, log = FALSE)
pSHASH(q, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
qSHASH(p, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
```

```
rSHASH(n, mu = 0, sigma = 1, nu = 0.5, tau = 0.5)

SHASHo(mu.link = "identity", sigma.link = "log", nu.link = "identity",
      tau.link = "log")
dSHASHo(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)
pSHASHo(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
qSHASHo(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
rSHASHo(n, mu = 0, sigma = 1, nu = 0, tau = 1)

SHASHo2(mu.link = "identity", sigma.link = "log", nu.link = "identity",
      tau.link = "log")
dSHASHo2(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)
pSHASHo2(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
qSHASHo2(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
rSHASHo2(n, mu = 0, sigma = 1, nu = 0, tau = 1)

LOGSHASH(mu.link = "identity", sigma.link = "log", nu.link = "log",
     tau.link = "log")
dLOGSHASH(x, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, log = FALSE)
pLOGSHASH(q, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
qLOGSHASH(p, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
rLOGSHASH(n, mu = 0, sigma = 1, nu = 0.5, tau = 0.5)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter. |
| tau.link | Defines the tau.link, with "log" link as the default for the tau parameter. |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of skewness nu parameter values |
| tau | vector of kurtosis tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The probability density function of the Sinh-Arcsinh distribution, SHASH, Jones(2005), is defined as

$$f(y|\mu,\sigma\,\nu,\tau) = \frac{c}{\sqrt{2\pi}\sigma(1+z^2)^{1/2}} e^{-r^2/2}$$

where

$$r = \frac{1}{2}\left\{\exp\left[\tau \sinh^{-1}(z)\right] - \exp\left[-\nu \sinh^{-1}(z)\right]\right\}$$

and

$$c = \frac{1}{2}\left\{\tau \exp\left[\tau \sinh^{-1}(z)\right] + \nu \exp\left[-\nu \sinh^{-1}(z)\right]\right\}$$

and $z = (y-\mu)/\sigma$ for $-\infty < y < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$, $\nu > 0$ and $\tau > 0$, see pp. 396-397 of Rigby et al. (2019).

The parameters $\mu$ and $\sigma$ are the location and scale of the distribution. The parameter $\nu$ determines the left hand tail of the distribution with $\nu > 1$ indicating a lighter tail than the normal and $\nu < 1$ heavier tail than the normal. The parameter $\tau$ determines the right hand tail of the distribution in the same way.

The second form of the Sinh-Arcsinh distribution can be found in Jones and Pewsey (2009, p.2) denoted by SHASHo and the probability density function is defined as,

$$f(y|\mu,\sigma,\nu,\tau) = \frac{\tau c}{\sigma\sqrt{2\pi}(1+z^2)^{1/2}} \exp\left(-\frac{1}{2}r^2\right)$$

where

$$r = \sinh(\tau\ \sinh^{-1}(z) - \nu)$$

and

$$c = \cosh(\tau \sinh^{-1}(z) - \nu)$$

and $z = (y-\mu)/\sigma$ for $-\infty < y < \infty$, $-\infty < \mu < +\infty$, $\sigma > 0$, $-\infty < \nu < +\infty$ and $\tau > 0$, see pp. 398-400 of Rigby et al. (2019)

The third form of the Sinh-Arcsinh distribution (Jones and Pewsey, 2009, p.8) divides the distribution by sigma for the density of the unstandardized variable. This distribution is denoted by SHASHo2 and has pdf

$$f(y|\mu,\sigma,\nu,\tau) = \frac{c}{\sigma}\frac{\tau}{\sqrt{2\pi}}\frac{1}{\sqrt{1+z^2}} - \exp -\frac{r^2}{2}$$

where $z = (y-\mu)/(\sigma\tau)$, with $r$ and $c$ as for the pdf of the SHASHo distribution, for $-\infty < y < \infty$, $\mu = (-\infty, +\infty)$, $\sigma > 0$, $\nu = (-\infty, +\infty)$ and $\tau > 0$.

## Value

SHASH() returns a gamlss.family object which can be used to fit the SHASH distribution in the gamlss() function. dSHASH() gives the density, pSHASH() gives the distribution function, qSHASH() gives the quantile function, and rSHASH() generates random deviates.

## Warning

The qSHASH and rSHASH are slow since they are relying on golden section for finding the quantiles

## Note

The parameters sigma, nu and tau are highly correlated. It is recomended to use method=mixed(10,100) for fitting.

## Author(s)

Bob Rigby, Mikis Stasinopoulos and Fiona McElduff

## References

Jones, M. C. (2005) p 546-547 in the discussion of Rigby, R. A. and Stasinopoulos D. M. (2005) *Appl. Statist.*, **54**, part 3.

Jones and Pewsey (2009) Sinh-arcsinh distributions. *Biometrika.* **96**(4), pp. 761?780.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, JSU, BCT

## Examples

```
SHASH()   #
plot(function(x)dSHASH(x, mu=0,sigma=1, nu=1, tau=2), -5, 5,
 main = "The SHASH  density mu=0,sigma=1,nu=1, tau=2")
plot(function(x) pSHASH(x, mu=0,sigma=1,nu=1, tau=2), -5, 5,
 main = "The BCPE  cdf mu=0, sigma=1, nu=1, tau=2")
dat<-rSHASH(100,mu=10,sigma=1,nu=1,tau=1.5)
hist(dat)
```

```
# library(gamlss)
# gamlss(dat~1,family=SHASH, control=gamlss.control(n.cyc=30))
```

---

SI *The Sichel dustribution for fitting a GAMLSS model*

---

## Description

The SI() function defines the Sichel distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dSI, pSI, qSI and rSI define the density, distribution function, quantile function and random generation for the Sichel SI(), distribution.

## Usage

```
SI(mu.link = "log", sigma.link = "log", nu.link = "identity")
dSI(x, mu = 0.5, sigma = 0.02, nu = -0.5, log = FALSE)
pSI(q, mu = 0.5, sigma = 0.02, nu = -0.5, lower.tail = TRUE,
       log.p = FALSE)
qSI(p, mu = 0.5, sigma = 0.02, nu = -0.5, lower.tail = TRUE,
    log.p = FALSE, max.value = 10000)
rSI(n, mu = 0.5, sigma = 0.02, nu = -0.5)
tofyS(y, mu, sigma, nu, what = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive mu |
| sigma | vector of positive despersion parameter |
| nu | vector of nu |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| max.value | a constant, set to the default value of 10000 for how far the algorithm should look for q |
| y | the y variable. The function tofyS() should be not used on its own. |
| what | take values 1 or 2, for function tofyS(). |

## Details

The probability function of the Sichel distribution is given by

$$f(y|\mu,\sigma,\nu) = \frac{\mu^y K_{y+\nu}(\alpha)}{y!(\alpha\sigma)^{y+\nu}K_\nu(\frac{1}{\sigma})}$$

where $\alpha^2 = \frac{1}{\sigma^2} + \frac{2\mu}{\sigma}$, for $y = 0, 1, 2, ..., \infty$ where $\mu > 0$ , $\sigma > 0$ and $-\infty < \nu < \infty$ and $K_\lambda(t) = \frac{1}{2}\int_0^\infty x^{\lambda-1}\exp\{-\frac{1}{2}t(x + x^{-1})\}dx$ is the modified Bessel function of the third kind. Note that the above parameterization is different from Stein, Zucchini and Juritz (1988) who use the above probability function but treat $\mu$, $\alpha$ and $\nu$ as the parameters. Note that $\sigma = [(\mu^2+\alpha^2)^{\frac{1}{2}} - \mu]^{-1}$. See also pp 510-511 of Rigby *et al.* (2019).

## Value

Returns a `gamlss.family` object which can be used to fit a Sichel distribution in the `gamlss()` function.

## Author(s)

Akantziliotou C., Rigby, R. A., Stasinopoulos D. M. and Marco Enea

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

Stein, G. Z., Zucchini, W. and Juritz, J. M. (1987). Parameter Estimation of the Sichel Distribution and its Multivariate Extension. *Journal of American Statistical Association*, **82**, 938-944.

(see also https://www.gamlss.com/).

## See Also

gamlss.family, PIG, NBI, NBII

## Examples

```
SI()# gives information about the default links for the  Sichel distribution
#plot the pdf using plot
plot(function(y) dSI(y, mu=10, sigma=1, nu=1), from=0, to=100, n=100+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=100),pSI(seq(from=0,to=100), mu=10, sigma=1, nu=1), type="h")   # cdf
# generate random sample
```

```
tN <- table(Ni <- rSI(100, mu=5, sigma=1, nu=1))
r <- barplot(tN, col='lightblue')
# fit a model to the data
# library(gamlss)
# gamlss(Ni~1,family=SI, control=gamlss.control(n.cyc=50))
```

---

SICHEL *The Sichel distribution for fitting a GAMLSS model*

---

## Description

The SICHEL() function defines the Sichel distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dSICHEL, pSICHEL, qSICHEL and rSICHEL define the density, distribution function, quantile function and random generation for the Sichel SICHEL(), distribution. The function VSICHEL gives the variance of a fitted Sichel model.

The functions ZASICHEL() and ZISICHEL() are the zero adjusted (hurdle) and zero inflated versions of the Sichel distribution, respectively. That is four parameter distributions.

The functions dZASICHEL, dZISICHEL, pZASICHEL,pZISICHEL, qZASICHEL qZISICHEL rZASICHEL and rZISICHEL define the probability, cumulative, quantile and random generation functions for the zero adjusted and zero inflated Sichel distributions, ZASICHEL(), ZISICHEL(), respectively.

## Usage

```
SICHEL(mu.link = "log", sigma.link = "log", nu.link = "identity")
dSICHEL(x, mu=1, sigma=1, nu=-0.5, log=FALSE)
pSICHEL(q, mu=1, sigma=1, nu=-0.5, lower.tail = TRUE,
         log.p = FALSE)
qSICHEL(p, mu=1, sigma=1, nu=-0.5,  lower.tail = TRUE,
        log.p = FALSE, max.value = 10000)
rSICHEL(n, mu=1, sigma=1, nu=-0.5, max.value = 10000)
VSICHEL(obj)
tofySICHEL(y, mu, sigma, nu)

ZASICHEL(mu.link = "log", sigma.link = "log", nu.link = "identity",
         tau.link = "logit")
dZASICHEL(x, mu = 1, sigma = 1, nu = -0.5, tau = 0.1, log = FALSE)
pZASICHEL(q, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
          lower.tail = TRUE, log.p = FALSE)
qZASICHEL(p, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
          lower.tail = TRUE, log.p = FALSE, max.value = 10000)
rZASICHEL(n, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
          max.value = 10000)

ZISICHEL(mu.link = "log", sigma.link = "log", nu.link = "identity",
          tau.link = "logit")
dZISICHEL(x, mu = 1, sigma = 1, nu = -0.5, tau = 0.1, log = FALSE)
```

```
pZISICHEL(q, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
         lower.tail = TRUE, log.p = FALSE)
qZISICHEL(p, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
         lower.tail = TRUE, log.p = FALSE, max.value = 10000)
rZISICHEL(n, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
         max.value = 10000)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter |
| tau.link | Defines the tau.link, with "logit" link as the default for the tau parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive mu |
| sigma | vector of positive dispersion parameter sigma |
| nu | vector of nu |
| tau | vector of probabilities tau |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| max.value | a constant, set to the default value of 10000 for how far the algorithm should look for q |
| obj | a fitted Sichel gamlss model |
| y | the y variable, the tofySICHEL() should not be used on its own. |

## Details

The probability function of the Sichel distribution SICHEL is given by

$$f(y|\mu,\sigma,\nu) = \frac{(\mu/b)^y K_{y+\nu}(\alpha)}{y!(\alpha\sigma)^{y+\nu}K_\nu(\frac{1}{\sigma})}$$

for $y = 0, 1, 2, ..., \infty$, $\mu > 0$ , $\sigma > 0$ and $-\infty < \nu < \infty$ where

$$\alpha^2 = \frac{1}{\sigma^2} + \frac{2\mu}{\sigma}$$

$$c = K_{\nu+1}(1/\sigma)/K_\nu(1/\sigma)$$

and $K_\lambda(t)$ is the modified Bessel function of the third kind see pp 508-510 of Rigby *et al.* (2019). The expected value in this parametrization is $\mu$. i.e.$E(y) = \mu$.

Note that the above parametrization is different from Stein, Zucchini and Juritz (1988) who use the above probability function but treat $\mu$, $\alpha$ and $\nu$ as the parameters.

The definition of the zero adjusted Sichel distribution, `ZASICHEL` and the the zero inflated Sichel distribution, `ZISICHEL`, are given in pp. 517-518 and pp. 519-520 of of Rigby *et al.* (2019), respectively.

## Value

Returns a `gamlss.family` object which can be used to fit a Sichel distribution in the `gamlss()` function.

## Author(s)

Rigby, R. A., Stasinopoulos D. M., Akantziliotou C and Marco Enea.

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

Stein, G. Z., Zucchini, W. and Juritz, J. M. (1987). Parameter Estimation of the Sichel Distribution and its Multivariate Extension. *Journal of American Statistical Association*, **82**, 938-944.

(see also https://www.gamlss.com/).

## See Also

`gamlss.family`, `PIG` , `SI`

## Examples

```
SICHEL()# gives information about the default links for the  Sichel distribution
#plot the pdf using plot
plot(function(y) dSICHEL(y, mu=10, sigma=1, nu=1), from=0, to=100, n=100+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=100),pSICHEL(seq(from=0,to=100), mu=10, sigma=1, nu=1), type="h")  # cdf
# generate random sample
tN <- table(Ni <- rSICHEL(100, mu=5, sigma=1, nu=1))
r <- barplot(tN, col='lightblue')
# fit a model to the data
```

```
# library(gamlss)
# gamlss(Ni~1,family=SICHEL, control=gamlss.control(n.cyc=50))
```

---

SIMPLEX *The simplex distribution for fitting a GAMLSS*

---

## Description

The functions SIMPLEX() define the simplex distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). SIMPLEX() has mean equal to the parameter mu and sigma as scale parameter, see below. The functions dSIMPLEX, pSIMPLEX qSIMPLEX and rSIMPLEX define the density, comulative distribution function, quantile function and random generation for the simplex distribution.

## Usage

```
SIMPLEX(mu.link = "logit", sigma.link = "log")
dSIMPLEX(x, mu = 0.5, sigma = 1, log = FALSE)
pSIMPLEX(q, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qSIMPLEX(p, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rSIMPLEX(n = 1, mu = 0.5, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | the mu link function with default logit |
| sigma.link | the sigma link function with default log |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The simplex distribution, SIMPLEX, is given as

$$f(y|\mu,\sigma) = \frac{1}{(2\pi\sigma^2 y^3(1-y)^3)^{1/2}} \exp(-\frac{1}{2\sigma^2}\frac{(y-\mu)^2}{y(1-y)\mu^2(1-\mu)^2})$$

for $0 < y < 1$, $0 < \mu < 1$ and $\sigma > 0$ see p 464 of Rigby et al. (2019).

## Value

SIMPLEX() returns a gamlss.family object which can be used to fit a simplex distribution in the gamlss() function.

## Author(s)

Bob Rigby, Mikis Stasinopoulos and Fernanda De Bastiani

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## Examples

```
SIMPLEX()#  default links for the simplex distribution
plot(function(y) dSIMPLEX(y, mu=.5 ,sigma=1), 0.001, .999)
plot(function(y) pSIMPLEX(y, mu=.5 ,sigma=1), 0.001, 0.999)
plot(function(y) qSIMPLEX(y, mu=.5 ,sigma=1), 0.001, 0.999)
plot(function(y) qSIMPLEX(y, mu=.5 ,sigma=1, lower.tail=FALSE), 0.001, .999)
```

---

# SN1 *Skew Normal Type 1 distribution for fitting a GAMLSS*

---

## Description

The function SN1() defines the Skew Normal Type 1 distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with parameters mu, sigma and nu. The functions dSN1, pSN1, qSN1 and rSN1 define the density, distribution function, quantile function and random generation for the SN1 parameterization of the Skew Normal Type 1 distribution.

## Usage

```
SN1(mu.link = "identity", sigma.link = "log", nu.link="identity")
dSN1(x, mu = 0, sigma = 1, nu = 0, log = FALSE)
pSN1(q, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)
qSN1(p, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)
rSN1(n, mu = 0, sigma = 1, nu = 0)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "'identity"' links the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "'log"' as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "'identity"' as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise P[X > x] |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the Skew Normal Type 1 distribution in the function SN1 is

$$f(y|\mu,\sigma,\nu) = \frac{2}{\sigma}\phi(z)\Phi(\nu z)$$

for $(-\infty < y < +\infty)$, $(-\infty < \mu < +\infty)$, $\sigma > 0$ and $(-\infty < \nu < +\infty)$ where $z = (y-\mu)/\sigma$ and $\phi()$ and $\Phi()$ are the pdf and cdf of the standard nornal distribution, respectively, see pp. 378-379 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a Skew Normal Type 1 distribution in the gamlss() function.

## Note

This is a special case of the Skew Exponential Power type 1 distribution (SEP1) where tau=2.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Fiona McElduff

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
par(mfrow=c(2,2))
y<-seq(-3,3,0.2)
plot(y, dSN1(y), type="l" , lwd=2)
q<-seq(-3,3,0.2)
plot(q, pSN1(q), ylim=c(0,1), type="l", lwd=2)
p<-seq(0.0001,0.999,0.05)
plot(p, qSN1(p), type="l", lwd=2)
dat <- rSN1(100)
hist(rSN1(100), nclass=30)
```

---

SN2 *Skew Normal Type 2 distribution for fitting a GAMLSS*

---

## Description

The function SN2() defines the Skew Normal Type 2 distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with parameters mu, sigma and nu. The functions dSN2, pSN2, qSN2 and rSN2 define the density, distribution function, quantile function and random generation for the SN2 parameterization of the Skew Normal Type 2 distribution.

## Usage

```
SN2(mu.link = "identity", sigma.link = "log", nu.link = "log")
dSN2(x, mu = 0, sigma = 1, nu = 2, log = FALSE)
pSN2(q, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
qSN2(p, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
rSN2(n, mu = 0, sigma = 1, nu = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "'identity"' links the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "'log"' as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "'log"' as the default for the sigma parameter |
| x, q | vector of quantiles |

| | |
|---|---|
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of scale parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise P[X > x] |
| p | vector of probabilities |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the Skew Normal Type 2 distribution in the function SN2 is

$$f(y|\mu,\sigma,\nu) = \frac{c}{\sigma}\exp\left[\frac{1}{2}(\nu z)^2\right]$$

if $y < \mu$

$$f(y|\mu,\sigma,\nu) = \frac{c}{\sigma}\exp\left[\frac{1}{2}(\frac{z}{\nu})^2\right]$$

if $y \geq \mu$

for $(-\infty < y < +\infty)$, $(-\infty < \mu < +\infty)$, $\sigma > 0$ and $\nu > 0$ where $z = (y-\mu)/\sigma$ and $c = \sqrt{2}\nu/\left[\sqrt{\pi}(1+\nu^2)\right]$ see pp. 380-381 of Rigby et al. (2019).

## Value

returns a gamlss.family object which can be used to fit a Skew Normal Type 2 distribution in the gamlss() function.

## Note

This is a special case of the Skew Exponential Power type 3 distribution (SEP3)where tau=2.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Fiona McElduff.

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
par(mfrow=c(2,2))
y<-seq(-3,3,0.2)
plot(y, dSN2(y), type="l" , lwd=2)
q<-seq(-3,3,0.2)
plot(q, pSN2(q), ylim=c(0,1), type="l", lwd=2)
p<-seq(0.0001,0.999,0.05)
plot(p, qSN2(p), type="l", lwd=2)
dat <- rSN2(100)
hist(rSN2(100), nclass=30)
```

---

# ST1 *The skew t distributions, type 1 to 5*

---

## Description

There are 5 different skew t distributions implemented in GAMLSS.

The Skew t type 1 distribution, ST1, is based on Azzalini (1986), see pp. 411-412 of Rigby et al. (2019).

The skew t type 2 distribution, ST2, is based on Azzalini and Capitanio (2003) see pp. 412-414 of Rigby et al. (2019).

The skew t type 3 , ST3 and ST3C, distribution is based Fernande and Steel (1998) see pp 413-415 of Rigby et al. (2019). The difference betwwen the ST3 and ST3C is that the first is written entirely in R while the second is in C.

The skew t type 4 distribution , ST4, is a spliced-shape distribution see see pp 413-415 of Rigby et al. (2019).

The skew t type 5 distribution , ST5, is Jones and Faddy (2003).

The SST is a reparametrised version of ST3 where sigma is the standard deviation of the distribution.

## Usage

```
ST1(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link="log")
dST1(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pST1(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
qST1(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
rST1(n, mu = 0, sigma = 1, nu = 0, tau = 2)

ST2(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log")
dST2(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pST2(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
qST2(p, mu = 1, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
```

```
rST2(n, mu = 0, sigma = 1, nu = 0, tau = 2)

ST3(mu.link = "identity", sigma.link = "log", nu.link = "log", tau.link = "log")
dST3(x, mu = 0, sigma = 1, nu = 1, tau = 10, log = FALSE)
pST3(q, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
qST3(p, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
rST3(n, mu = 0, sigma = 1, nu = 1, tau = 10)

ST3C(mu.link = "identity", sigma.link = "log", nu.link = "log", tau.link = "log")
dST3C(x, mu = 0, sigma = 1, nu = 1, tau = 10, log = FALSE)
pST3C(q, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
qST3C(p, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
rST3C(n, mu = 0, sigma = 1, nu = 1, tau = 10)

SST(mu.link = "identity", sigma.link = "log", nu.link = "log",
   tau.link = "logshiftto2")
dSST(x, mu = 0, sigma = 1, nu = 0.8, tau = 7, log = FALSE)
pSST(q, mu = 0, sigma = 1, nu = 0.8, tau = 7, lower.tail = TRUE, log.p = FALSE)
qSST(p, mu = 0, sigma = 1, nu = 0.8, tau = 7, lower.tail = TRUE, log.p = FALSE)
rSST(n, mu = 0, sigma = 1, nu = 0.8, tau = 7)

ST4(mu.link = "identity", sigma.link = "log", nu.link = "log", tau.link = "log")
dST4(x, mu = 0, sigma = 1, nu = 1, tau = 10, log = FALSE)
pST4(q, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
qST4(p, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
rST4(n, mu = 0, sigma = 1, nu = 1, tau = 10)

ST5(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log")
dST5(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)
pST5(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)
qST5(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)
rST5(n, mu = 0, sigma = 1, nu = 0, tau = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "$1/mu^2$" and "log" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse" and "identity" |
| nu.link | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "$1/mu^2$" and "log" |
| tau.link | Defines the nu.link, with "log" link as the default for the nu parameter. Other links are "inverse", "identity" |
| x, q | vector of quantiles |
| mu | vector of mu parameter values |
| sigma | vector of scale parameter values |
| nu | vector of nu parameter values |

| | |
|---|---|
| tau | vector of tau parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The definitions of all Skew *t* distributions is given in pp.409-420 of of Rigby et al. (2019).

## Note

The mean of the ex-Gaussian is $\mu + \nu$ and the variance is $\sigma^2 + \nu^2$.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Azzalini A. (1986) Futher results on a class of distributions which includes the normal ones, *Statistica*, **46**, pp. 199-208.

Azzalini A. and Capitanio, A. Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution, *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **65**, pp. 367-389.

Jones, M.C. and Faddy, M. J. (2003) A skew extension of the t distribution, with applications. *Journal of the Royal Statistical Society*, Series B, **65**, pp 159-174.

Fernandez, C. and Steel, M. F. (1998) On Bayesian modeling of fat tails and skewness. *Journal of the American Statistical Association*, **93**, pp. 359-371.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, SEP1, SHASH

## Examples

```
y<- rST5(200, mu=5, sigma=1, nu=.1)
hist(y)
curve(dST5(x, mu=30 ,sigma=5,nu=-1), -50, 50, main = "The ST5  density mu=30 ,sigma=5,nu=1")
# library(gamlss)
# m1<-gamlss(y~1, family=ST1)
# m2<-gamlss(y~1, family=ST2)
# m3<-gamlss(y~1, family=ST3)
# m4<-gamlss(y~1, family=ST4)
# m5<-gamlss(y~1, family=ST5)
# GAIC(m1,m2,m3,m4,m5)
```

---

# TF *t family distribution for fitting a GAMLSS*

---

## Description

The function TF defines the t-family distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dTF, pTF, qTF and rTF define the density, distribution function, quantile function and random generation for the specific parameterization of the t distribution given in details below, with mean equal to $\mu$ and standard deviation equal to $\sigma(\frac{\nu}{\nu-2})^{0.5}$ with the degrees of freedom $\nu$ The function TF2 is a different parametrization where sigma is the standard deviation.

## Usage

```
TF(mu.link = "identity", sigma.link = "log", nu.link = "log")
dTF(x, mu = 0, sigma = 1, nu = 10, log = FALSE)
pTF(q, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
qTF(p, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
rTF(n, mu = 0, sigma = 1, nu = 10)

TF2(mu.link = "identity", sigma.link = "log", nu.link = "logshiftto2")
dTF2(x, mu = 0, sigma = 1, nu = 10, log = FALSE)
pTF2(q, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
qTF2(p, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
rTF2(n, mu = 0, sigma = 1, nu = 10)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "identity" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "log" link as the default for the nu parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |

| | |
|---|---|
| nu | vector of the degrees of freedom parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

Definition file for t family distribution TF():

$$f(y|\mu,\sigma,\nu) = \frac{1}{\sigma B(1/2,\nu/2))\nu^{0.5}}\left[1+\frac{(y-\mu)^2}{\nu\sigma^2}\right]^{-(\nu+1)/2}$$

for $-\infty < y < +\infty$, $-\infty < \mu < +\infty$, $\sigma > 0$ and $\nu > 0$ see pp. 382-383 of Rigby et al. (2019). Note that $z = (y-\mu)/\sigma$ has a standard t distribution with degrees of freedom $\nu$ see pp. 382-383 of Rigby et al. (2019).

Definition file for t family distribution TF2():

$$f(y|\mu,\sigma,\nu) = \frac{1}{\sigma B(1/2,\nu/2)(\nu-2)^{0.5}}\left[1+\frac{(y-\mu)^2}{(\nu-2)\sigma^2}\right]^{-(\nu+1)/2}$$

for $-\infty < y < +\infty$, $-\infty < \mu < +\infty$, $\sigma > 0$ and $\nu > 2$ see pp. 382-383 of Rigby et al. (2019). Note that $z = (y-\mu)/\sigma$ has a standard t distribution with degrees of freedom $\nu$ see pp. 383-384 of Rigby et al. (2019).

## Value

TF() returns a gamlss.family object which can be used to fit a t distribution in the gamlss() function. dTF() gives the density, pTF() gives the distribution function, qTF() gives the quantile function, and rTF() generates random deviates. The latest functions are based on the equivalent R functions for gamma distribution.

## Note

$\mu$ is the mean and $\sigma[\nu/(\nu-2)]^{0.5}$ is the standard deviation of the t family distribution. $\nu > 0$ is a positive real valued parameter.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Kalliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

## See Also

gamlss.family

## Examples

```
TF()# gives information about the default links for the t-family distribution
# library(gamlss)
#data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=TF, data=abdom) # fits
#plot(h)
newdata<-rTF(1000,mu=0,sigma=1,nu=5) # generates 1000 random observations
hist(newdata)
```

---

# WARING *Waring distribution for fitting a GAMLSS model*

---

## Description

The function WARING() defines the Waring distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with mean equal to the parameter mu and scale parameter sigma. The functions dWARING, pWARING, qWARING and rWARING define the density, distribution function, quantile function and random generation for the WARING parameterization of the Waring distribution.

## Usage

```
WARING(mu.link = "log", sigma.link = "log")
dWARING(x, mu = 2, sigma = 2, log = FALSE)
pWARING(q, mu = 2, sigma = 2, lower.tail = TRUE, log.p = FALSE)
qWARING(p, mu = 2, sigma = 2, lower.tail = TRUE, log.p = FALSE,
    max.value = 10000)
rWARING(n, mu = 2, sigma = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| x | vector of (non-negative integer) quantiles. |

| | |
|---|---|
| q | vector of quantiles. |
| p | vector of probabilities. |
| n | number of random values to return. |
| mu | vector of positive mu values. |
| sigma | vector of positive sigma values. |
| lower.tail | logical; if TRUE (default) probabilities are $P[Y \leq y]$, otherwise, $P[Y > y]$. |
| log, log.p | logical; if TRUE probabilities p are given as log(p). |
| max.value | constant; generates a sequence of values for the cdf function. |

## Details

The Waring distribution, WARING, has density,

$$f(y|\mu,\sigma) = \frac{B(y+\mu\sigma^{-1}, \sigma^{-1}+2)}{B(\mu\sigma^{-1}, \sigma^{-1}+1)}$$

for $y = 0, 1, 2, \ldots$, $\mu > 0$ and $\sigma > 0$ with $E(y) = \mu$, see pp. 490-492 of Rigby *et al.* (2019).

## Value

Returns a gamlss.family object which can be used to fit a Waring distribution in the gamlss() function.

## Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos. <f.mcelduff@ich.ucl.ac.uk>

## References

Wimmer, G. and Altmann, G. (1999) *Thesaurus of univariate discrete probability distributions.* Stamm.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family

## Examples

```
par(mfrow=c(2,2))
y<-seq(0,20,1)
plot(y, dWARING(y), type="h")
q <- seq(0, 20, 1)
plot(q, pWARING(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qWARING(p), type="s")
dat <- rWARING(100)
hist(dat)
#summary(gamlss(dat~1, family=WARING))
```

---

WEI *Weibull distribution for fitting a GAMLSS*

---

## Description

The function WEI can be used to define the Weibull distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). [Note that the GAMLSS function WEI2 uses a different parameterization for fitting the Weibull distribution.] The functions dWEI, pWEI, qWEI and rWEI define the density, distribution function, quantile function and random generation for the specific parameterization of the Weibul distribution.

## Usage

```
WEI(mu.link = "log", sigma.link = "log")
dWEI(x, mu = 1, sigma = 1, log = FALSE)
pWEI(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qWEI(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rWEI(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse", "identity" and "own" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse", "identity" and "own" |
| x, q | vector of quantiles |
| mu | vector of the mu parameter |
| sigma | vector of sigma parameter |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the function WEI is given by

$$f(y|\mu,\sigma) = \frac{\sigma y^{\sigma-1}}{\mu^{\sigma}} \exp\left[-\left(\frac{y}{\mu}\right)^{\sigma}\right]$$

for $y > 0$, $\mu > 0$ and $\sigma > 0$ see pp. 435-436 of Rigby et al. (2019). The GAMLSS functions dWEI, pWEI, qWEI, and rWEI can be used to provide the pdf, the cdf, the quantiles and random generated numbers for the Weibull distribution with argument mu, and sigma. [See the GAMLSS function WEI2 for a different parameterization of the Weibull.]

## Value

WEI() returns a gamlss.family object which can be used to fit a Weibull distribution in the gamlss() function. dWEI() gives the density, pWEI() gives the distribution function, qWEI() gives the quantile function, and rWEI() generates random deviates. The latest functions are based on the equivalent R functions for Weibull distribution.

## Note

The mean in WEI is given by $\mu\Gamma(\frac{1}{\sigma}+1)$ and the variance $\mu^2\left[\Gamma(\frac{2}{\sigma}+1)-(\Gamma(\frac{1}{\sigma}+1))^2\right]$

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, WEI2, WEI3

## Examples

```
WEI()
dat<-rWEI(100, mu=10, sigma=2)
# library(gamlss)
# gamlss(dat~1, family=WEI)
```

WEI2 *A specific parameterization of the Weibull distribution for fitting a GAMLSS*

## Description

The function WEI2 can be used to define the Weibull distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). This is the parameterization of the Weibull distribution usually used in proportional hazard models and is defined in details below. [Note that the GAMLSS function WEI uses a different parameterization for fitting the Weibull distribution.] The functions dWEI2, pWEI2, qWEI2 and rWEI2 define the density, distribution function, quantile function and random generation for the specific parameterization of the Weibull distribution.

## Usage

```
WEI2(mu.link = "log", sigma.link = "log")
dWEI2(x, mu = 1, sigma = 1, log = FALSE)
pWEI2(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qWEI2(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rWEI2(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse" and "identity" |
| x, q | vector of quantiles |
| mu | vector of the mu parameter values |
| sigma | vector of sigma parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the function WEI2 is given by

$$f(y|\mu,\sigma) = \sigma\mu y^{\sigma-1} e^{-\mu y^{\sigma}}$$

for $y > 0$, $\mu > 0$ and $\sigma > 0$, see pp. 436-437 of Rigby et al. (2019). The GAMLSS functions dWEI2, pWEI2, qWEI2, and rWEI2 can be used to provide the pdf, the cdf, the quantiles and random generated numbers for the Weibull distribution with argument mu, and sigma. [See the GAMLSS function WEI for a different parameterization of the Weibull.]

## Value

WEI2() returns a gamlss.family object which can be used to fit a Weibull distribution in the gamlss() function. dWEI2() gives the density, pWEI2() gives the distribution function, qWEI2() gives the quantile function, and rWEI2() generates random deviates. The latest functions are based on the equivalent R functions for Weibull distribution.

## Warning

In WEI2 the estimated parameters mu and sigma can be highly correlated so it is advisable to use the CG() method for fitting [as the RS() method can be veru slow in this situation.]

## Note

The mean in WEI2 is given by $\mu^{-1/\sigma}\Gamma(\frac{1}{\sigma}+1)$ and the variance $\mu^{-2/\sigma}(\Gamma(\frac{2}{\sigma}+1)-\left[\Gamma(\frac{1}{\sigma}+1)\right]^2)$

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, WEI,WEI3,

## Examples

```
WEI2()
dat<-rWEI(100, mu=.1, sigma=2)
hist(dat)
# library(gamlss)
# gamlss(dat~1, family=WEI2, method=CG())
```

WEI3 *A specific parameterization of the Weibull distribution for fitting a GAMLSS*

## Description

The function WEI3 can be used to define the Weibull distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). This is a parameterization of the Weibull distribution where $\mu$ is the mean of the distribution. [Note that the GAMLSS functions WEI and WEI2 use different parameterizations for fitting the Weibull distribution.] The functions dWEI3, pWEI3, qWEI3 and rWEI3 define the density, distribution function, quantile function and random generation for the specific parameterization of the Weibull distribution.

## Usage

```
WEI3(mu.link = "log", sigma.link = "log")
dWEI3(x, mu = 1, sigma = 1, log = FALSE)
pWEI3(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qWEI3(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rWEI3(n, mu = 1, sigma = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity" |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse" and "identity" |
| x, q | vector of quantiles |
| mu | vector of the mu parameter values |
| sigma | vector of sigma parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |

## Details

The parameterization of the function WEI3 is given by

$$f(y|\mu,\sigma) = \frac{\sigma}{\beta}\left(\frac{y}{\beta}\right)^{\sigma-1} e^{-\left(\frac{y}{\beta}\right)^{\sigma}}$$

where $\beta = \frac{\mu}{\Gamma((1/\sigma)+1)}$ for $y > 0$, $\mu > 0$ and $\sigma > 0$ see pp. 437-438 of Rigby et al. (2019). The GAMLSS functions dWEI3, pWEI3, qWEI3, and rWEI3 can be used to provide the pdf, the cdf, the quantiles and random generated numbers for the Weibull distribution with argument mu, and sigma. [See the GAMLSS function WEI for a different parameterization of the Weibull.]

## Value

WEI3() returns a gamlss.family object which can be used to fit a Weibull distribution in the gamlss() function. dWEI3() gives the density, pWEI3() gives the distribution function, qWEI3() gives the quantile function, and rWEI3() generates random deviates. The latest functions are based on the equivalent R functions for Weibull distribution.

## Warning

In WEI3 the estimated parameters mu and sigma can be highly correlated so it is advisable to use the CG() method for fitting [as the RS() method can be very slow in this situation.]

## Note

The mean in WEI3 is given by $\mu$ and the variance $\mu^2 \left\{\Gamma(2/\sigma+1)/\left[\Gamma(1/\sigma+1)\right]^2 - 1\right\}$ see pp. 438 of Rigby et al. (2019)

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, WEI, WEI2

## Examples

```
WEI3()
dat<-rWEI(100, mu=.1, sigma=2)
# library(gamlss)
# gamlss(dat~1, family=WEI3, method=CG())
```

YULE *Yule distribution for fitting a GAMLSS model*

## Description

The function YULE defines the Yule distribution, a one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with mean equal to the parameter mu. The functions dYULE, pYULE, qYULE and rYULE define the density, distribution function, quantile function and random generation for the YULE parameterization of the Yule distribution.

## Usage

```
YULE(mu.link = "log")
dYULE(x, mu = 2, log = FALSE)
pYULE(q, mu = 2, lower.tail = TRUE, log.p = FALSE)
qYULE(p, mu = 2, lower.tail = TRUE, log.p = FALSE,
    max.value = 10000)
rYULE(n, mu = 2)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| x | vector of (non-negative integer) quantiles. |
| q | vector of quantiles. |
| p | vector of probabilities. |
| n | number of random values to return. |
| mu | vector of positive mu values. |
| lower.tail | logical; if TRUE (default) probabilities are $P[Y \leq y]$, otherwise, $P[Y > y]$. |
| log, log.p | logical; if TRUE probabilities p are given as log(p). |
| max.value | constant; generates a sequence of values for the cdf function. |

## Details

The Yule distribution has density

$$P(Y = y|\mu) = (\mu^{-1} + 1)B(y + 1, \mu^{-1} + 2)$$

for $y = 0, 1, 2, \ldots$ and $\mu > 0$, see pp 477-478 of Rigby et al. (2019).

## Value

Returns a gamlss.family object which can be used to fit a Yule distribution in the gamlss() function.

### Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos.

### References

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

Wimmer, G. and Altmann, G. (1999) *Thesaurus of univariate discrete probability distributions.* Stamm.

(see also https://www.gamlss.com/).

### See Also

gamlss.family

### Examples

```
par(mfrow=c(2,2))
y<-seq(0,20,1)
plot(y, dYULE(y), type="h")
q <- seq(0, 20, 1)
plot(q, pYULE(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qYULE(p), type="s")
dat <- rYULE(100)
hist(dat)
#summary(gamlss(dat~1, family=YULE))
```

---

## ZABB — *Zero inflated and zero adjusted Binomial distribution for fitting in GAMLSS*

---

### Description

The function ZIBB defines the zero inflated beta binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIBB, pZIBB, qZIBB and rZINN define the density, distribution function, quantile function and random generation for the zero inflated beta binomial, ZIBB, distribution.

The function ZABB defines the zero adjusted beta binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZABB, pZABB, qZABB and rZABB define the density, distribution function, quantile function and random generation for the zero inflated beta binomial, ZABB(), distribution.

## Usage

```
ZABB(mu.link = "logit", sigma.link = "log", nu.link = "logit")
ZIBB(mu.link = "logit", sigma.link = "log", nu.link = "logit")

dZIBB(x, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1, log = FALSE)
dZABB(x, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1, log = FALSE)

pZIBB(q, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)
pZABB(q, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)

qZIBB(p, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)
qZABB(p, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)

rZIBB(n, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1)
rZABB(n, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog"'(complementary log-log) |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link | Defines the sigma.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog"'(complementary log-log) |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive probabilities |
| sigma | vector of positive dispertion parameter |
| nu | vector of positive probabilities |
| bd | vector of binomial denominators |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

The definition of the zero adjusted beta binomial distribution, ZABB and the the zero inflated beta binomial distribution, ZIBB, are given in p. 527 and p. 528 of of Rigby *et al.* (2019), respectively.

## Value

The functions ZIBB and ZABB return a gamlss.family object which can be used to fit a zero inflated or zero adjusted beta binomial distribution respectively in the gamlss() function.

### Author(s)

Mikis Stasinopoulos, Bob Rigby

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07..

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

### See Also

gamlss.family, NBI, NBII

### Examples

```
ZIBB()
ZABB()
# creating data and plotting them
 dat <- rZIBB(1000, mu=.5, sigma=.5, nu=0.1, bd=10)
   r <- barplot(table(dat), col='lightblue')
dat1 <- rZABB(1000, mu=.5, sigma=.2, nu=0.1, bd=10)
   r1 <- barplot(table(dat1), col='lightblue')
```

---

## ZABI *Zero inflated and zero adjusted Binomial distribution for fitting in GAMLSS*

---

### Description

The ZABI() function defines the zero adjusted binomial distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZABI, pZABI, qZABI and rZABI define the density, distribution function, quantile function and random generation for the zero adjusted binomial, ZABI(), distribution.

The ZIBI() function defines the zero inflated binomial distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIBI, pZIBI, qZIBI and rZIBI define the density, distribution function, quantile function and random generation for the zero inflated binomial, ZIBI(), distribution.

## Usage

```
 ZABI(mu.link = "logit", sigma.link = "logit")
dZABI(x, bd = 1, mu = 0.5, sigma = 0.1, log = FALSE)
pZABI(q, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZABI(p, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZABI(n, bd = 1, mu = 0.5, sigma = 0.1)

 ZIBI(mu.link = "logit", sigma.link = "logit")
dZIBI(x, bd = 1, mu = 0.5, sigma = 0.1, log = FALSE)
pZIBI(q, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZIBI(p, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZIBI(n, bd = 1, mu = 0.5, sigma = 0.1)
```

## Arguments

| | |
|---|---|
| `mu.link` | Defines the `mu.link`, with "logit" link as the default for the `mu` parameter. Other links are "probit" and "cloglog"'(complementary log-log) |
| `sigma.link` | Defines the `sigma.link`, with "logit" link as the default for the `mu` parameter. Other links are "probit" and "cloglog"'(complementary log-log) |
| `x` | vector of (non-negative integer) quantiles |
| `mu` | vector of positive probabilities |
| `sigma` | vector of positive probabilities |
| `bd` | vector of binomial denominators |
| `p` | vector of probabilities |
| `q` | vector of quantiles |
| `n` | number of random values to return |
| `log, log.p` | logical; if TRUE, probabilities p are given as log(p) |
| `lower.tail` | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

The definition of the zero adjusted binomial distribution, `ZABI` and the the zero inflated binomial distribution, `ZIBI`, are given in p. 526 and p. 527 of of Rigby *et al.* (2019), respectively.

## Value

The functions `ZABI` and `ZIBI` return a `gamlss.family` object which can be used to fit a binomial distribution in the `gamlss()` function.

## Note

The response variable should be a matrix containing two columns, the first with the count of successes and the second with the count of failures.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BI

## Examples

```
ZABI()
curve(dZABI(x, mu = .5, bd=10), from=0, to=10, n=10+1, type="h")
tN <- table(Ni <- rZABI(1000, mu=.2, sigma=.3, bd=10))
r <- barplot(tN, col='lightblue')

ZIBI()
curve(dZIBI(x, mu = .5, bd=10), from=0, to=10, n=10+1, type="h")
tN <- table(Ni <- rZIBI(1000, mu=.2, sigma=.3, bd=10))
r <- barplot(tN, col='lightblue')
```

---

ZAGA *The zero adjusted Gamma distribution for fitting a GAMLSS model*

---

## Description

The function ZAGA() defines the zero adjusted Gamma distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The zero adjusted Gamma distribution is similar to the Gamma distribution but allows zeros as y values. The extra parameter nu models the probabilities at zero. The functions dZAGA, pZAGA, qZAGA and rZAGA define the density, distribution function, quartile function and random generation for the ZAGA parameterization of the zero adjusted Gamma distribution. plotZAGA can be used to plot the distribution. meanZAGA calculates the expected value of the response for a fitted model.

## Usage

```
ZAGA(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZAGA(x, mu = 1, sigma = 1, nu = 0.1, log = FALSE)
pZAGA(q, mu = 1, sigma = 1, nu = 0.1, lower.tail = TRUE,
      log.p = FALSE)
qZAGA(p, mu = 1, sigma = 1, nu = 0.1, lower.tail = TRUE,
       log.p = FALSE)
rZAGA(n, mu = 1, sigma = 1, nu = 0.1, ...)
plotZAGA(mu = 5, sigma = 1, nu = 0.1, from = 0, to = 10,
          n = 101,  main=NULL, ...)
meanZAGA(obj)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "logit" link as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of probability at zero parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| from | where to start plotting the distribution from |
| to | up to where to plot the distribution |
| obj | a fitted gamlss object |
| main | for title in the plot |
| ... | ... can be used to pass the uppr.limit argument to qIG |

## Details

The Zero adjusted GA distribution is given as

$$f(y|\mu,\sigma\,\nu) = \nu$$

if (y=0)

$$f(y|\mu,\sigma,\nu) = (1-\nu)\left[\frac{1}{(\sigma^2\mu)^{1/\sigma^2}}\ \frac{y^{\frac{1}{\sigma^2}-1}\ e^{-y/(\sigma^2\mu)}}{\Gamma(1/\sigma^2)}\right]$$

otherwise

for $y = (0,\infty)$, $\mu > 0$, $\sigma > 0$ and $0 < \nu < 1$. $E(y) = (1-\nu)\mu$ and $Var(y) = (1-\nu)\mu^2(\nu+\sigma^2)$.

## Value

The function ZAGA returns a gamlss.family object which can be used to fit a zero adjusted Gamma distribution in the gamlss() function.

## Author(s)

Bob Rigby, Mikis Stasinopoulos and Almond Stocker

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, GA, ZAIG

## Examples

```
ZAGA()# gives information about the default links for the ZAGA distribution
# plotting the function
PPP <- par(mfrow=c(2,2))
plotZAGA(mu=1, sigma=.5, nu=.2, from=0,to=3)
#curve(dZAGA(x,mu=1, sigma=.5, nu=.2), 0,3) # pdf
curve(pZAGA(x,mu=1, sigma=.5, nu=.2), 0,3,  ylim=c(0,1)) # cdf
curve(qZAGA(x,mu=1, sigma=.5, nu=.2), 0,.99) # inverse cdf
y<-rZAGA(100, mu=1, sigma=.5, nu=.2) # randomly generated values
hist(y)
par(PPP)
# check that the  positive part sums up to .8 (since nu=0.2)
integrate(function(x) dZAGA(x,mu=1, sigma=.5, nu=.2), 0,Inf)
```

ZAIG *The zero adjusted Inverse Gaussian distribution for fitting a GAMLSS model*

## Description

The function ZAIG() defines the zero adjusted Inverse Gaussian distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The zero adjusted Inverse Gaussian distribution is similar to the Inverse Gaussian distribution but allows zeros as y values. The extra parameter models the probabilities at zero. The functions dZAIG, pZAIG, qZAIG and rZAIG define the density, distribution function, quantile function and random generation for the ZAIG parameterization of the zero adjusted Inverse Gaussian distribution. plotZAIG can be used to plot the distribution. meanZAIG calculates the expected value of the response for a fitted model.

## Usage

```
ZAIG(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZAIG(x, mu = 1, sigma = 1, nu = 0.1, log = FALSE)
pZAIG(q, mu = 1, sigma = 1, nu = 0.1, lower.tail = TRUE, log.p = FALSE)
qZAIG(p, mu = 1, sigma = 1, nu = 0.1, lower.tail = TRUE, log.p = FALSE)
rZAIG(n, mu = 1, sigma = 1, nu = 0.1, ...)
plotZAIG(mu = 5, sigma = 1, nu = 0.1, from = 0, to = 10, n = 101,
         main = NULL,...)
meanZAIG(obj)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the nu.link, with "logit" link as the default for the sigma parameter |
| x, q | vector of quantiles |
| mu | vector of location parameter values |
| sigma | vector of scale parameter values |
| nu | vector of probability at zero parameter values |
| log, log.p | logical; if TRUE, probabilities p are given as log(p). |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| p | vector of probabilities. |
| n | number of observations. If length(n) > 1, the length is taken to be the number required |
| from | where to start plotting the distribution from |
| to | up to where to plot the distribution |
| obj | a fitted BEINF object |
| main | for title in the plot |
| ... | ... can be used to pass the uppr.limit argument to qIG |

## Details

The Zero adjusted IG distribution is given as

$$f(y|\mu,\sigma\,\nu) = \nu$$

if (y=0)

$$f(y|\mu,\sigma,\nu) = (1-\nu)\frac{1}{\sqrt{2\pi\sigma^2 y^3}}\exp(-\frac{(y-\mu)^2}{2\mu^2\sigma^2 y})$$

otherwise

for $y = (0,\infty)$, $\mu > 0$, $\sigma > 0$ and $0 < \nu < 1$. $E(y) = (1-\nu)\mu$ and $Var(y) = (1-\nu)\mu^2(\nu+\mu\sigma^2)$.

## Value

returns a `gamlss.family` object which can be used to fit a zero adjusted inverse Gaussian distribution in the `gamlss()` function.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Heller, G. Stasinopoulos M and Rigby R.A. (2006) The zero-adjusted Inverse Gaussian distribution as a model for insurance claims. in *Proceedings of the 21th International Workshop on Statistial Modelling*, eds J. Hinde, J. Einbeck and J. Newell, pp 226-233, Galway, Ireland.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, IG

## Examples

```
ZAIG()# gives information about the default links for the ZAIG distribution
# plotting the distribution
plotZAIG( mu =10 , sigma=.5, nu = 0.1, from = 0, to=10, n = 101)
# plotting the cdf
plot(function(y) pZAIG(y, mu=10 ,sigma=.5, nu = 0.1 ), 0, 1)
```

```
# plotting the inverse cdf
plot(function(y) qZAIG(y, mu=10 ,sigma=.5, nu = 0.1 ), 0.001, .99)
# generate random numbers
dat <- rZAIG(100,mu=10,sigma=.5, nu=.1)
# fit a model to the data
# library(gamlss)
# m1<-gamlss(dat~1,family=ZAIG)
# meanZAIG(m1)[1]
```

---

ZANBI *Zero inflated and zero adjusted negative binomial distributions for fitting a GAMLSS model*

---

## Description

The function ZINBI defines the zero inflated negative binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZINBI, pZINBI, qZINBI and rZINBI define the density, distribution function, quantile function and random generation for the zero inflated negative binomial, ZINBI(), distribution.

The function ZANBI defines the zero adjusted negative binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZANBI, pZANBI, qZANBI and rZANBI define the density, distribution function, quantile function and random generation for the zero inflated negative binomial, ZANBI(), distribution.

## Usage

```
ZINBI(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZINBI(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZINBI(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZINBI(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
rZINBI(n, mu = 1, sigma = 1, nu = 0.3)
ZANBI(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZANBI(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZANBI(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZANBI(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
rZANBI(n, mu = 1, sigma = 1, nu = 0.3)
```

## Arguments

| | |
|---|---|
| mu.link | Defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link | Defines the mu.link, with "logit" link as the default for the nu parameter |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive means |
| sigma | vector of positive despersion parameter |

| | |
|---|---|
| nu | vector of zero probability parameter |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

The definition of the zero adjusted Negative Binomial type I distribution, ZANBI and the the zero inflated Negative Binomial type I distribution, ZINBI, are given in p. 512 and pp. 513-514 of of Rigby *et al.* (2019), respectively.

## Value

The functions ZINBI and ZANBI return a gamlss.family object which can be used to fit a zero inflated or zero adjusted Negative Binomial type I distribution respectively in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, NBI, NBII

## Examples

```
ZINBI()
ZANBI()
# creating data and plotting them
 dat <- rZINBI(1000, mu=5, sigma=.5, nu=0.1)
   r <- barplot(table(dat), col='lightblue')
dat1 <- rZANBI(1000, mu=5, sigma=.5, nu=0.1)
   r1 <- barplot(table(dat1), col='lightblue')
```

ZAP *Zero adjusted poisson distribution for fitting a GAMLSS model*

## Description

The function ZAP defines the zero adjusted Poisson distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZAP, pZAP, qZAP and rZAP define the density, distribution function, quantile function and random generation for the inflated poisson, ZAP(), distribution.

## Usage

```
ZAP(mu.link = "log", sigma.link = "logit")
dZAP(x, mu = 5, sigma = 0.1, log = FALSE)
pZAP(q, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZAP(p, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZAP(n, mu = 5, sigma = 0.1)
```

## Arguments

| | |
|---|---|
| mu.link | defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | defines the sigma.link, with "logit" link as the default for the sigma parameter which in this case is the probability at zero. Other links are "probit" and "cloglog"'(complementary log-log) |
| x | vector of (non-negative integer) |
| mu | vector of positive means |
| sigma | vector of probabilities at zero |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

Details about the zero adjusted Poison, ZAP can be found pp 494-496 of Rigby *et al.* (2019).

## Value

The function ZAP returns a gamlss.family object which can be used to fit a zero inflated poisson distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/)..

## See Also

gamlss.family, PO, ZIP, ZIP2, ZALG

## Examples

```
ZAP()
# creating data and plotting them
dat<-rZAP(1000, mu=5, sigma=.1)
r <- barplot(table(dat), col='lightblue')
```

---

ZIP *Zero inflated poisson distribution for fitting a GAMLSS model*

---

## Description

The function ZIP defines the zero inflated Poisson distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIP, pZIP, qZIP and rZIP define the density, distribution function, quantile function and random generation for the inflated poisson, ZIP(), distribution.

## Usage

```
ZIP(mu.link = "log", sigma.link = "logit")
dZIP(x, mu = 5, sigma = 0.1, log = FALSE)
pZIP(q, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZIP(p, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZIP(n, mu = 5, sigma = 0.1)
```

## Arguments

| | |
|---|---|
| mu.link | defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | defines the sigma.link, with "logit" link as the default for the sigma parameter which in this case is the probability at zero. Other links are "probit" and "cloglog"'(complementary log-log) |
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive means |
| sigma | vector of probabilities at zero |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

Let $Y = 0$ with probability $\sigma$ and $Y \sim Po(\mu)$ with probability $(1 - \sigma)$ the Y has a Zero inflated Poisson Distribution given by

$$f(y) = \sigma + (1 - \sigma)e^{-\mu}$$

if (y=0)

$$f(y) = (1 - \sigma)\frac{e^{-\mu}\mu^{y}}{y!}$$

if (y>0) for $y = 0, 1, ...$ see pp 498-500 of Rigby *et al.* (2019). The expected values in this parametrization is $E(y) = (1 - \sigma)\mu$.

## Value

returns a gamlss.family object which can be used to fit a zero inflated poisson distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Lambert, D. (1992), Zero-inflated Poisson Regression with an application to defects in Manufacturing, *Technometrics*, **34**, pp 1-14.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape,(with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi:10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi:10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, PO, ZIP2

## Examples

```
ZIP()# gives information about the default links for the normal distribution
# creating data and plotting them
dat<-rZIP(1000, mu=5, sigma=.1)
r <- barplot(table(dat), col='lightblue')
# library(gamlss)
# fit the distribution
# mod1<-gamlss(dat~1, family=ZIP)# fits a constant for mu and sigma
# fitted(mod1)[1]
# fitted(mod1,"sigma")[1]
```

---

# ZIP2 *Zero inflated poisson distribution for fitting a GAMLSS model*

---

## Description

The function ZIP2 defines the zero inflated Poisson type 2 distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIP2, pZIP2, qZIP2 and rZIP2 define the density, distribution function, quantile function and random generation for the inflated poisson, ZIP2(), distribution. The ZIP2 is a different parameterization of the ZIP distribution. In the ZIP2 the mu is the mean of the distribution.

## Usage

```
ZIP2(mu.link = "log", sigma.link = "logit")
dZIP2(x, mu = 5, sigma = 0.1, log = FALSE)
pZIP2(q, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZIP2(p, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZIP2(n, mu = 5, sigma = 0.1)
```

## Arguments

| | |
|---|---|
| mu.link | defines the mu.link, with "log" link as the default for the mu parameter |
| sigma.link | defines the sigma.link, with "logit" link as the default for the sigma parameter which in this case is the probability at zero. Other links are "probit" and "cloglog"'(complementary log-log) |

| | |
|---|---|
| x | vector of (non-negative integer) quantiles |
| mu | vector of positive means |
| sigma | vector of probabilities at zero |
| p | vector of probabilities |
| q | vector of quantiles |
| n | number of random values to return |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |

## Details

The parametrization used for this version of the zero inflated Poison distribution ZIP2 can be found in pp 500-501 of Rigby *et al.* (2019). Note that the mean of the distribution in this parameterization is $\mu$.

## Value

returns a gamlss.family object which can be used to fit a zero inflated poisson distribution in the gamlss() function.

## Author(s)

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## References

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(see also https://www.gamlss.com/).

## See Also

gamlss.family, ZIP

## Examples

```
ZIP2()# gives information about the default links for the normal distribution
# creating data and plotting them
dat<-rZIP2(1000, mu=5, sigma=.1)
r <- barplot(table(dat), col='lightblue')
# fit the disteibution
# library(gamlss)
# mod1<-gamlss(dat~1, family=ZIP2)# fits a constant for mu and sigma
# fitted(mod1)[1]
# fitted(mod1,"sigma")[1]
```

---

ZIPF *The zipf and zero adjusted zipf distributions for fitting a GAMLSS model*

---

## Description

This function ZIPF() defines the zipf distribution, Johnson et. al., (2005), sections 11.2.20, p 527-528. The zipf distribution is an one parameter distribution with long tails (a discete version of the Pareto distrbution). The function ZIPF() creates a gamlss.family object to be used in GAMLSS fitting. The functions dZIPF, pZIPF, qZIPF and rZIPF define the density, distribution function, quantile function and random generation for the zipf, ZIPF(), distribution. The function zetaP() defines the zeta function and it is based on the zeta function defined on the VGAM package of Thomas Yee, see Yee (2017).

The distribution zipf is defined on $y = 1, 2, 3, \ldots, \infty$, the zero adjusted zipf permits values on $y =, 01, 2, \ldots, \infty$. The function ZAZIPF() defines the zero adjusted zipf distribution. The function ZAZIPF() creates a gamlss.family object to be used in GAMLSS fitting. The functions dZAZIPF, pZAZIPF, qZAZIPF and rZAZIPF define the density, distribution function, quantile function and random generation for the zero adjusted zipf, ZAZIPF(), distribution.

## Usage

```
ZIPF(mu.link = "log")
dZIPF(x, mu = 1, log = FALSE)
pZIPF(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qZIPF(p, mu = 1, lower.tail = TRUE, log.p = FALSE,
       max.value = 10000)
rZIPF(n, mu = 1, max.value = 10000)
zetaP(x)
ZAZIPF(mu.link = "log", sigma.link = "logit")
dZAZIPF(x, mu = 0.5, sigma = 0.1, log = FALSE)
pZAZIPF(q, mu = 0.5, sigma = 0.1, lower.tail = TRUE,
        log.p = FALSE)
qZAZIPF(p, mu = 0.5, sigma = 0.1, lower.tail = TRUE,
       log.p = FALSE, max.value = 10000)
rZAZIPF(n, mu = 0.5, sigma = 0.1, max.value = 10000)
```

## Arguments

| | |
|---|---|
| mu.link | the link function for the parameter mu with default log |
| x, q | vectors of (non-negative integer) quantiles |
| p | vector of probabilities |
| mu | vector of positive parameter |
| log, log.p | logical; if TRUE, probabilities p are given as log(p) |
| lower.tail | logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x] |
| n | number of random values to return |
| max.value | a constant, set to the default value of 10000, It is used in the q function which numerically calculates how far the algorithm should look for q. Maybe for zipf data the values has to increase at a considerable computational cost. |
| sigma.link | the link function for the parameter aigma with default logit |
| sigma | a vector of probabilities of zero |

## Details

The probability density for the zipf distribution, ZIPF, is:

$$f(y|\mu) = \frac{y^{-(\mu+1)}}{\zeta(\mu+1)}$$

for $y = 1, 2, \ldots, \infty$, $\mu > 0$ and where $\zeta() = \sum_i^n i^{-b}$ is the (Reimann) zeta function.

The distribution has mean $\zeta(\mu)/\zeta(\mu+1)$ and variance $\zeta(\mu+1)\zeta(\mu-1) - [\zeta(\mu)]^2/[\zeta(\mu+1)]^2$, see pp 479-480 of Rigby et al. (2019)

For more details about the zero-adjusted Zipf distributions, ZAZIPF, see see pp 496-498 of Rigby *et al.* (2019).

## Value

The function ZIPF() returns a gamlss.family object which can be used to fit a zipf distribution in the gamlss() function.

## Note

Because the zipf distrbution has very long tails the max.value in the q and r, may need to increase.

## Author(s)

Mikis Stasinopoulos and Bob Rigby

## References

N. L. Johnson, A. W. Kemp, and S. Kotz. (2005) Univariate Discrete Distributions. Wiley, New York, 3rd edition.

Thomas W. Yee (2017). VGAM: Vector Generalized Linear and Additive Models. R package version 1.0-3. https://CRAN.R-project.org/package=VGAM

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(see also https://www.gamlss.com/).

## See Also

PO, LG, GEOM, YULE

## Examples

```
# ZIPF
par(mfrow=c(2,2))
y<-seq(1,20,1)
plot(y, dZIPF(y), type="h")
q <- seq(1, 20, 1)
plot(q, pZIPF(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qZIPF(p), type="s")
dat <- rZIPF(100)
hist(dat)
# ZAZIPF
y<-seq(0,20,1)
plot(y, dZAZIPF(y,  mu=.9, sigma=.1), type="h")
q <- seq(1, 20, 1)
plot(q, pZAZIPF(q,  mu=.9, sigma=.1), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p, qZAZIPF(p,  mu=.9, sigma=.1), type="s")
dat <- rZAZIPF(100, mu=.9, sigma=.1)
hist(dat)
```

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